# A general iterative approach for the system-level joint optimization of pavement maintenance, rehabilitation, and reconstruction planning

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**Abstract**

We formulate a general bottom-up model for the joint optimization of maintenance, rehabilitation, and reconstruction (MR&R) schedules for a system of heterogeneous pavement segments under budget constraints. The objective is to minimize the total costs incurred to both the highway users and the pavement management agency. We propose a Lagrange multiplier approach together with derivative-free quasi-Newton algorithms to solve the problem for two scenarios: i) with a combined budget constraint for all the treatments; and ii) with one budget constraint for each treatment. The system-level solution approach has the following merits: i) it can be applied to problems with any forms of segment-level models for user and agency costs, deterioration process, and treatment effectiveness, given that the solution to the segment-level problem is available; ii) under the combined budget constraint, it ensures that the optimality gap of the system-level solution is bounded by a term that depends upon the optimality gap of the segment-level solutions; and iii) it exhibits linear complexity with the number of segments.

At the segment level, a new maintenance effectiveness model fitted on empirical data is proposed and incorporated into the MR&R optimization program. A greedy heuristic algorithm is developed, which greatly reduces the computation time without compromising the solution quality. Combining the system-level and segment-level models and solution algorithms, we examine a batch of numerical cases. The results show considerable cost savings from the incorporation of maintenance, and from jointly optimizing the use of a combined agency budget. A number of managerial insights stemmed from the numerical case studies are discussed, which can help highway agencies formulate more cost-efficient MR&R plans and budget allocation.

Keywords: System-level MR&R planning, Budget constraint, Preventive maintenance model, Lagrange multiplier, Quasi-Newton methods.

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# 1. Introduction

## *1.1. Background*

Surface roads constitute the world's largest transportation infrastructure network. For example, the United States alone has over 4 million miles of roads, which served over 3 trillion vehicle-miles in the year of 2015 (CBO, 2016; ASCE, 2017). The constantly increasing amount of vehicle-miles creates ever-growing pavement deterioration and aging in many regions over the world. The deteriorated pavements in turn incur higher costs for vehicle repair, traffic congestion, and extra fuel consumption and emission, among others. This imposes a great challenge for highway agencies to optimally plan MR&R activities for the road pavements, especially given that a large portion of the pavements are currently in poor conditions, and that the available annual budget rises consistently slower than the MR&R costs needed (ASCE, 2017).

Conventionally, a highway agency's long-term planning decision considered only rehabilitation and reconstruction activities. However, in recent decades many studies have reported the sizable effects of preventive maintenance activities (e.g. chip seal, microsurfacing) on slowing down the pavement's deterioration and extending its service life (Chong, 1989; Ponniah and Kennepohl, 1996; Labi and Sinha, 2003; Mamlouk and Dosa, 2014). These cheap maintenance treatments are particularly attractive for highway agencies under budget pressure. However, most highway agencies do not have well-established preventive maintenance planning mechanism (Peshkin et al., 2004). Hence, an optimization model for the joint planning of not only the rehabilitation and reconstruction activities, but also the preventive maintenance activities, is highly desired. Unfortunately such a model is missing in the literature to the best of our knowledge. We next examine the strength and deficiency of existing studies in the realm of MR&R planning optimization.

## *1.2. Literature review*

Studies in this realm commenced by optimizing the rehabilitation planning of a single segment (Friesz and Fernandez, 1979; Fernandez and Friesz, 1981; Markow and Balta, 1985). A variety of segment-level optimization models have thenceforth been developed, which are characterized by the pavement deterioration process (memoryless or history-dependent), the number of treatments, and whether the time and/or pavement states are discrete or continuous variables. Table 1 summarizes the modeling features and solution approaches of select segment-level studies. Of note is that the table shows a general trend of evolution from simpler models (with memoryless deterioration process, single treatment, and discrete variables) to more complicated but realistic ones (with history-dependent deterioration process, multiple treatments, and continuous variables). This is partly thanks to the development of more sophisticated approaches for seeking global optimal solutions, e.g. calculus of variation (Ouyang and Madanat, 2006; Lee and Madanat, 2014b). The most complicated (and realistic) segment-level model so far seems to be Lee and Madanat (2014a), which optimized the planning of all the three treatments (maintenance, rehabilitation and reconstruction) with history-dependent deterioration process.[1] However, the solution relied on the technique of approximate dynamic programming, which requires high computation time and thus may not be suitable for large-scale systems of pavements. Another finding is that the solution approaches in Table 1 are usually problem-specific. This means a solution approach generally cannot be applied, without making substantial changes, to solve a different version of the segment-level optimization model (e.g. with different deterioration process, number of treatments, or treatment effectiveness models). Finally, the maintenance effectiveness models used in segment-level MR&R optimization are unrealistic. For example, the maintenance model used by Gu et al. (2012) and Lee and Madanat (2014a, b) was hypothesized with ungrounded parameter values. As a result, the optimal MR&R plan obtained by Lee and Madanat (2014a) showed greater deterioration rate reduction could occur when maintenance was applied to a pavement near the end of its lifecycle (see Fig. 4a of the cited work), which contradicts with the common understanding in practice.

On the other hand, a highway agency often manages hundreds of pavement segments or more. Thus they are more interested in models that can jointly optimize for a system of pavement segments under certain budget constraints, which can be incorporated into their pavement management systems. However, the system-level problems are by nature more complicated than the segment-level ones. This is why a smaller number of studies were found in this category, including some works that relied on the highly idealized "top-down" approaches (Kuhn and Madanat, 2005; Durango-Cohen and Sarutipand, 2007). Those top-down models assumed homogeneous pavement segments in a system, and are thus unrealistic and unsuitable for real-world implementation.

The more realistic, "bottom-up" approaches that appreciate the heterogeneity in pavement segments have also been applied to system-level MR&R planning optimization. A number of select bottom-up studies are summarized in Table 2. The table shows that many of the cited studies relied on metaheuristic methods (e.g. genetic algorithm and tabu search), especially for the models involving multiple treatments and history-dependent deterioration process. Metaheuristic methods are known to be unable to guarantee the global optimality of the solution (Blum and Roli, 2003). In fact they are often unable to assess how close the solution is to the global optimum.[2] Some other works also sought to optimizing Lagrangian and Lagrangian dual functions of the original problem (Sathaye and Madanat, 2011; 2012; Lee et al., 2016). However, these works often relied on the convexity of the problem formulation to obtain optimal solutions. Unfortunately, the convexity is not always

[1] Although Lee and Madanat (2014b) also optimized three treatments, it assumed a constant effectiveness of maintenance (in terms of the reduction in pavement deterioration rate), while Lee and Madanat (2014a) relaxed this assumption.

[2] Since the global optimum is not available, a common means to assess the quality of a heuristic solution is to compare that against a lower bound of the global optimum (for minimization problems). However, such a lower bound is often unavailable too.

guaranteed, given the fact that the empirical models for pavement deterioration and treatment effectiveness may vary from case to case. Another problem of most existing bottom-up studies is that the solution approaches are highly dependent upon the segment-level empirical models[3]; i.e., they cannot be directly applied to another system-level problem with different segment-level models. This is undesirable since there are many variants of segment-level models (see again Table 1), and new empirical models may arise in the future to replace the present ones. Finally, to the authors' best knowledge, there is no system-level optimization model for the joint planning of three or more treatments (including preventive maintenance) with sufficient realistic features.[4] This perhaps is because the existing approaches listed in Table 2 are insufficient to find optimal solutions within acceptable computation time when more treatments are considered. Note that incorporating preventive maintenance into the optimal MR&R planning would add much to the complexity of the problem, partly because the effect of maintenance on the pavements is very different from that of rehabilitation and reconstruction (Mamlouk and Dosa, 2014).

### *1.3. The research question and rundown of the paper*

Given the research gap in the literature revealed above, in this paper we will develop a computationally efficient and not problem-specific approach to find globally-optimal or near-optimal MR&R policies for large-scale pavement systems. To this end, we first propose a general formulation of the system-level problem that is independent of any specific segment-level models. Two scenarios are considered in the formulation: i) where a combined budget constraint is applied to all the MR&R treatments; and ii) where each treatment is subject to a separate budget constraint. A general solution approach is then developed to decompose the system-level problem into a number of segment-level subproblems. This is done by relaxing the budget constraint(s) via Lagrange multiplier(s). The optimization program is then converted to a bi-level one where the lower level is the segment-level subproblems which are solved by model-specific algorithms, and the upper level is to find the value of the Lagrange multiplier. We show for the combined-budget-constraint scenario that global optimality is retained at the system level via certain derivative-free iterative methods; i.e., if the segment-level subproblems are solved at or near optimality, then the global optimality or near-optimality of the system-level problem is guaranteed. Note that this is true regardless of whether the original problem is convex or not. Also note that the system-level approach can be applied regardless of the form of

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[3] The only exception is the approach used by Lee et al. (2016), which does not rely on the segment-level model specifics.

[4] Lee and Madanat (2015) and Lee et al. (2016) optimized for rehabilitation and reconstruction treatments only. Although Chu and Chen (2012) considered three treatments (fog seal, overlay, and reconstruction), they assumed overly-simplified treatment effectiveness models and searched for suboptimal (threshold-based) MR&R policies. Note that the threshold-based policy, while optimal for the problems with the simple, memoryless deterioration process (Ouyang and Madanat, 2006), has been proved to be suboptimal at the segment level when the history-dependent deterioration process is used (Lee and Madanat, 2014b).

segment-level models.

Table 1. Select studies on segment-level optimization of MR&R planning

| Model | Deterioration process | Number of treatments | Discrete/Continuous time or pavement state | Solution approach |
|---|---|---|---|---|
| Golabi et al. (1982) | memoryless | 1 | discrete | linear programming |
| Carnahan et al. (1987) | memoryless | 1 | discrete | dynamic programming |
| Fwa et al. (1994) | memoryless | 1 | discrete | genetic algorithm |
| Durango-Cohen (2007) | memoryless | 1 | hybrid[5] | dynamic programming |
| Friesz and Fernandez (1979) | memoryless | 1 | continuous | optimal control |
| Fernandez and Friesz (1981) | memoryless | 1 | continuous | optimal control |
| Tsunokawa and Schofer (1994) | memoryless | 1 | continuous | optimal control with trend curve approximation |
| Li and Madanat (2002) | memoryless | 1 | continuous | using the memoryless property |
| Ouyang and Madanat (2006) | memoryless | 1 | continuous | calculus of variation |
| Madanat (1993) | memoryless | 3 | discrete | dynamic programming |
| Madanat and Ben-Akiva (1994) | memoryless | 3 | discrete | dynamic programming |
| Gu et al. (2012) | memoryless | 2 | continuous | numerical method based on Ouyang and Madanat (2006)'s result |
| Rashid and Tsunokawa (2012) | memoryless | 3 | continuous | optimal control with trend curve approximation |
| Tsunokawa and Ul-Isalm (2002) | history-dependent | 1 | discrete | exhaustive search |
| Tsunokawa et al. (2006) | history-dependent | 1 | discrete | gradient search |
| Deshpande et al. (2010) | history-dependent | 1 | discrete | multi-objective genetic algorithm (MOGA) |
| Bai et al. (2015) | history-dependent | 1 | hybrid | dynamic programming |
| Miyamoto et al. (2000) | history-dependent | 2 | discrete | genetic algorithm |
| Lee and Madanat (2014a) | history-dependent | 3 | hybrid | dynamic programming |
| Lee and Madanat (2014b) | history-dependent | 3 | continuous | calculus of variation |

We propose a segment-level model that incorporates the history-dependent deterioration process and all the three types of treatments (preventive maintenance, rehabilitation, and reconstruction). A realistic maintenance effectiveness model is developed using the recent empirical data reported in the literature (Mamlouk and Dosa, 2014) to replace the hypothetical, flawed one that was used previously, and the new model produces reasonable results in the optimal MR&R plans. We also propose a greedy heuristic algorithm that reduces the computation time by 97% without compromising the solution quality (as compared against the dynamic programming algorithm used in

[5] The word "hybrid" in Tables 1 and 2 means "discrete time and continuous pavement state". The "hybrid" models are essentially discrete models since the solution space is discretized by adopting a discrete time scale.

the literature). The segment-level model and the solution algorithm are integrated with the general system-level approach to obtain optimal MR&R policies for pavement systems.

Table 2. Select studies on bottom-up system-level optimization of MR&R planning

| Study | Deterioration process | Number of treatments | Discrete/Continuous time or pavement state | Solution approach |
|---|---|---|---|---|
| Chan et al. (1994) | memoryless | 1 | discrete | genetic algorithm |
| Ouyang and Madanat (2004) | memoryless | 1 | hybrid | branch and bound; greedy heuristic |
| Ouyang (2007) | memoryless | 1 | hybrid | approximate dynamic programming |
| Hajibabai et al. (2014) | memoryless | 1 | hybrid | Lagrangian relaxation |
| Sathaye and Madanat (2011) | memoryless | 1 | continuous | Lagrange method |
| Sathaye and Madanat (2012) | memoryless | 1 | continuous | Lagrange dual method |
| Fwa et al. (1996) | memoryless | 2 | discrete | genetic algorithm |
| Chu and Chen (2012) | history-dependent | 3 | hybrid | tabu search |
| Lee and Madanat (2015) | history-dependent | 2 | hybrid | genetic algorithm |
| Lee et al. (2016) | history-dependent | 2 | discrete | Lagrange dual method |
| | | | | |

The models and solution approach are tested through a large number of numerical experiments. The results unveil many useful insights regarding how budget and other key operating parameters affect the optimal system-level MR&R policy. The numerical experiments also manifest the computational efficiency of our solution approach. Particularly, the computation time increases linearly with the size of the pavement system.

The rest of this paper is organized as follows: Section 2 presents the general formulation of the system-level problem and a general solution approach; Section 3 describes the details of the segment-level model and its solution approach; numerical case studies are furnished in Section 4; the insights, limitations, and future extensions of this paper are discussed in Section 5.

## 2. General formulation and solution approach for the system-level optimization of MR&R planning

A general formulation of the system-level MR&R planning problem, regardless of the segment-level models, is presented in Section 2.1. An iterative solution approach built upon the Lagrange Multiplier method is described in Section 2.2.

### *2.1. A general formulation*

The objective of the problem is to minimize the sum of the discounted user and agency costs, $Z_k$, for all the pavement segments $k \in \{1,2,\dots,K\}$ over a given planning horizon $T$ ($T = \infty$ denotes an infinite-horizon problem), as shown in (1a) below. For each segment $k$, $Z_k$ is a function of a vector of state variables (e.g. roughness level and age), denoted by $\boldsymbol{q}_k$, and a vector of management decision variables (e.g., timing and intensities of MR&R activities), $\boldsymbol{x}_k$. Note that the elements of $\boldsymbol{q}_k$ and $\boldsymbol{x}_k$ can be discrete or continuous functions of time. The $Z_k$ consists of the costs incurred to the users, $C_k^U$, and to the management agency, $\sum_{p=1}^{P} C_{kp}$, where $p \in \{1,\dots,P\}$ is the index of a treatment to be planned (i.e., maintenance, rehabilitation, and reconstruction).

Segment-specific constraints include the pavement deterioration model, treatment effectiveness models, initial pavement conditions, etc. These are divided into two classes: equality constraints (1b) and inequality constraints (1c), where $\boldsymbol{\Phi}_k$ and $\boldsymbol{\Psi}_k$ are again vectors of discrete or continuous functions of time. These constraints specify the pavements' initial conditions, how each pavement's state evolves over time (i.e. the deterioration process), and how each treatment may change the pavement's state, depending on the type, time and intensity of the treatment (i.e. the treatment effectiveness models). Finally, we present two versions of budget constraints in (1d-e): i) a combined budget that applies to the sum of agency costs for all the treatments across all segments, and ii) a number of separate budgets that each applies to a specific treatment. Note that $B$ and $B_p$ denote the annual combined budget and separate budget for treatment $p$, respectively; and $r$ is the annual discount factor. Note that here we assume the budget can be transferred across years over the planning horizon. This assumption was adopted by a number of previous studies (e.g. Sathaye and Madanat, 2011; 2012).

$$\min \sum_{k=1}^{K} Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = \sum_{k=1}^{K}\left(C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)\right) \tag{1a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0, \text{for } k = 1,\dots,K \tag{1b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \le 0, \text{for } k = 1,\dots,K \tag{1c}$$

$$\begin{cases} \text{combined budget:} & \frac{r}{1-e^{-rT}} \sum_{k=1}^{K} \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \le B & \text{(1d)} \\ \text{separate budgets:} & \frac{r}{1-e^{-rT}} \sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \le B_p \text{ for } p = 1,\cdots,P & \text{(1e)} \end{cases}$$

We next present an iterative approach for solving the above mathematical program.

### *2.2. An iterative approach using Lagrange Multipliers*

The segment-level formulation corresponding to the above system-level formulation is given in (2a-c). In the following discussion of this section, we assume that the solution to this segment-level problem has been developed a priori. This segment-level solution will be used as a building block in our proposed approach.

For each $k = 1,\dots,K$

$$\min\ Z_k(\boldsymbol{q}_k,\boldsymbol{x}_k) = C_k^U(\boldsymbol{q}_k,\boldsymbol{x}_k) + \sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k) \tag{2a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k,\boldsymbol{x}_k) = 0 \tag{2b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k,\boldsymbol{x}_k) \le 0 \tag{2c}$$

To be accurate, we describe the solution approach for the problems with the combined budget constraint (Section 2.2.1) and separate budget constraints (Section 2.2.2) one by one. However, they follow the same logic: first, the system-level problem is decomposed into $K$ segment-level subproblems, each having the form of (2a-c); and second, built upon the solutions to the segment-level subproblems, a gradient-free iterative algorithm is used to solve the system-level optimization problem.

*2.2.1. Combined-budget-constraint problem*

We introduce a Lagrange multiplier, $\lambda$, to relax the combined budget constraint (1d). The relaxed optimization is presented as follows:

$$\min\quad L(\boldsymbol{q},\boldsymbol{x},\lambda) = \sum_{k=1}^{K} Z_k(\boldsymbol{q}_k,\boldsymbol{x}_k) + \lambda\left(\sum_{k=1}^{K}\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k) - \frac{B}{r}(1-e^{-rT})\right) =$$

$$\sum_{k=1}^{K} H_k(\boldsymbol{q}_k,\boldsymbol{x}_k,\lambda) - \lambda\frac{B}{r}(1-e^{-rT}) \tag{3a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k,\boldsymbol{x}_k) = 0, \text{ for } k=1,\dots,K \tag{3b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k,\boldsymbol{x}_k) \le 0, \text{ for } k=1,\dots,K \tag{3c}$$

$$\begin{cases} \text{either: } \lambda = 0 \text{ and } V(0) = \sum_{k=1}^{K}\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k) - \frac{B}{r}(1-e^{-rT}) \le 0 \\ \text{or: } \quad \lambda > 0 \text{ and } V(\lambda) = \sum_{k=1}^{K}\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k) - \frac{B}{r}(1-e^{-rT}) = 0 \end{cases} \tag{3d}$$

where $L$ is the partial Lagrange function, and $H_k(\boldsymbol{q}_k,\boldsymbol{x}_k,\lambda) \equiv Z_k(\boldsymbol{q}_k,\boldsymbol{x}_k) + \lambda\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k)$. Constraint (3d) is the complementary slackness condition of optimality: $\lambda > 0$ when the budget constraint is binding, and $\lambda = 0$ otherwise. One can easily verify that the optimal solution of (3a-d) is always optimal to (1a-d); i.e., the relaxed program (3a-d) constructs a sufficient condition for the optimality of (1a-d).

Without constraint (3d), the remaining mathematical program (3a-c) can be decomposed by segment number $k$ as follows:

For each $k = 1,\dots,K$,

$$\min\ H_k(\boldsymbol{q}_k,\boldsymbol{x}_k,\lambda) = Z_k(\boldsymbol{q}_k,\boldsymbol{x}_k) + \lambda\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k)$$

$$= C_k^U(\boldsymbol{q}_k,\boldsymbol{x}_k) + (1+\lambda)\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k)$$

$$= C_k^U(\boldsymbol{q}_k,\boldsymbol{x}_k) + \sum_{p=1}^{P} \bar{C}_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k,\lambda) \tag{4a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k,\boldsymbol{x}_k) = 0 \tag{4b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k,\boldsymbol{x}_k) \le 0 \tag{4c}$$

where $\bar{C}_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k,\lambda) = (1+\lambda)C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k)$ can be considered as a "weighted" agency cost for treatment $p$ applied to segment $k$ (where the weight is $1+\lambda$). Note that for a given $\lambda$, $H_k$ has the same form as $Z_k$ with only a different weight for agency costs. Thus the solution to the segment-level problem (2a-c) can be readily applied to (4a-c) for each $k$ with a given $\lambda$. Note that if the global optimality of segment-level solutions is guaranteed, then the global optimality of the system-level problem is attained if a $\lambda$ is found to satisfy the complementary slackness condition (3d). Further, the following lemma ensures that if the segment-level solution is near-optimal (i.e., its relative cost gap from the optimal solution is bounded by a small fraction), then the resulting system-level solution is also near-optimal.

**Lemma 1**. For a given $\lambda$, suppose $\boldsymbol{x}_k^*(\lambda)$ is the exact optimal solution to the subproblem of segment $k$ $(k=1,2,\cdots,K)$, and $\boldsymbol{x}_k^H(\lambda)$ is a heuristic solution that satisfies:

$$\begin{cases} \left|C_k\big(\boldsymbol{x}_k^*(\lambda)\big)-C_k\left(\boldsymbol{x}_k^H(\lambda)\right)\right| \le \delta_1 \\ \left|Z_k\big(\boldsymbol{x}_k^*(\lambda)\big)-Z_k\left(\boldsymbol{x}_k^H(\lambda)\right)\right| \le \delta_2 \end{cases}, \forall k=1,2,\cdots,K, \lambda \ge 0 \tag{5}$$

where $C_k(\boldsymbol{x}_k)=\sum_{p=1}^{P} C_{kp}(\boldsymbol{q}_k,\boldsymbol{x}_k)$. Further assume $\lambda^*$ and $\lambda^H$ are the Lagrange multiplier values when the exact and the heuristic solutions are used, respectively; i.e.,

$$\lambda^* \cdot \left(\sum_{k=1}^{K} C_k\big(\boldsymbol{x}_k^*(\lambda^*)\big)-B\right)=0 \tag{6a}$$

$$\lambda^H \cdot \left(\sum_{k=1}^{K} C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)-B\right)=0 \tag{6b}$$

Then we have:

$$\left|\sum_{k=1}^{K} Z_k\big(\boldsymbol{x}_k^*(\lambda^*)\big)-\sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \le K\cdot(\max\{\lambda^*,\lambda^H\}\,\delta_1+\delta_2) \tag{7}$$

A sketched proof of Lemma 1 is furnished in Appendix A. Note (7) ensures that the percentage cost gap of the system-level problem is in the same magnitude of the percentage cost gaps of the segment-level heuristics, given that $\lambda^*$ and $\lambda^H$ are small.[6]

Finally, the following lemma specifies that as long as such a $\lambda$ exists, we are always able to find it via a properly designed Quasi-Newton algorithm. The proof of this lemma is furnished in Appendix B.

**Lemma 2**. $V(\lambda)$ is a (strictly) decreasing function of $\lambda$ if each segment-level problem furnishes a unique optimal solution.

An immediate corollary of this lemma is that there exists a unique solution of $\lambda$ to (3d) (as

[6] In our numerical case studies, $\lambda^H$ is always less than 3. The $\lambda^*$ is comparable to $\lambda^H$ in most cases since $|V(\lambda^H)-V(\lambda^*)| \le K\delta_1$. Exception may arise only when $B$ is near the maximum annual budget needed, where $V(\lambda)$ becomes flat.

long as program (3a-d) is feasible), and this solution can be attained by a number of iterative methods, including Newton's and Quasi-Newton methods (which presumably converge much faster than the methods of bisection, golden-section, etc.). Since the calculation of derivatives is often difficult and computationally inefficient due to the complicated mathematical forms of MR&R cost and effectiveness models, we next present an algorithm using a derivative-free method (modified secant method). In the following algorithm, $\delta$ denotes the tolerance level that is sufficiently small to guarantee the algorithm converges. The convergence of the algorithm is proved in Appendix C.[7]

**Algorithm 1:**

*Step 1*. Set $\lambda = \lambda^0 = 0$; solve the segment-level subproblems (4a-c) for each $k$. Evaluate $V(\lambda^0)$. If $V(\lambda^0) \leq 0$, end; otherwise go to Step 2.

*Step 2*. Select another initial value $\lambda = \lambda^1 > 0$, solve (4a-c) for each $k$ and evaluate $V(\lambda^1)$. If $|V(\lambda^1)| < \delta$, end; otherwise set $n = 1$ and go to Step 3.

*Step 3*. Set $\lambda = \lambda^{n+1} = \lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})}$. Solve (4a-c) for each $k$ and evaluate $V(\lambda^{n+1})$. If $|V(\lambda^{n+1})| < \delta$, end; otherwise, go to Step 4.

*Step 4*. If $V(\lambda^n) \cdot V(\lambda^{n+1}) > 0$ and $V(\lambda^{n-1}) \cdot V(\lambda^{n+1}) < 0$, set $\lambda^n = \lambda^{n-1}$. Set $n = n + 1$ and repeat Step 3.

### 2.2.2. *Separate-budget-constraint problem*

Similarly, we use a Lagrange multiplier, $\lambda_p$, to relax each of the $P$ budget constraints in (1e). The Lagrange function becomes:

$$\min \quad L(\boldsymbol{q}, \boldsymbol{x}, \lambda) = \sum_{k=1}^{K} Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P} \lambda_p \left( \sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B_p}{r}(1 - e^{-rT}) \right) =$$

$$\sum_{k=1}^{K} H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \boldsymbol{\lambda}) - \sum_{p=1}^{P} \lambda_p \frac{B_p}{r}(1 - e^{-rT}) \tag{8}$$

where $H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \boldsymbol{\lambda}) \equiv C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P}(1 + \lambda_p) C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k)$, and $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_P]^T$. The corresponding segment-level problem can be written as follows:

For each $k = 1, \ldots, K$

$$\min \ H_k(\boldsymbol{q}_k, \boldsymbol{x}_k, \boldsymbol{\lambda}) = C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \sum_{p=1}^{P}(1 + \lambda_p) C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \tag{9a}$$

$$\text{subject to: } \boldsymbol{\Phi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = 0 \tag{9b}$$

$$\boldsymbol{\Psi}_k(\boldsymbol{q}_k, \boldsymbol{x}_k) \leq 0 \tag{9c}$$

The complementary slackness conditions are:

$$\text{For } p = 1, \cdots, P, \begin{cases} \text{either: } \lambda_p = 0 \text{ and } V_p(\boldsymbol{\lambda}) = \sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B_p}{r}(1 - e^{-rT}) \leq 0 \\ \text{or: } \quad \lambda_p > 0 \text{ and } V_p(\boldsymbol{\lambda}) = \sum_{k=1}^{K} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) - \frac{B_p}{r}(1 - e^{-rT}) = 0 \end{cases} \tag{10}$$

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[7] Note that the original secant method cannot guarantee the convergence to the global optimum.

Similar to the combined-budget-constraint problem, the optimal solution of the relaxed program above is also optimal to the original problem under separate budget constraints. Here we propose a modified Broyden's method to formulate the following algorithm for solving the relaxed program.[8]

**Algorithm 2:**

*Step 1.* Set $\boldsymbol{\lambda} = \boldsymbol{\lambda}^0 \equiv [\lambda_1^0, \lambda_2^0, \dots, \lambda_P^0]^T = \mathbf{0} \equiv [0,0,\dots,0]^T$; solve the segment-level subproblems (9a-c) for each $k$. Evaluate $\boldsymbol{V}(\boldsymbol{\lambda}^0) = [V_1, \dots, V_P]^T$. If $\boldsymbol{V}(\boldsymbol{\lambda}^0) \le \mathbf{0}$, end; otherwise go to Step 2.

*Step 2.* Calculate the initial $P \times P$ Jacobian matrix $J^0$. For each $p = 1, \dots, P$, define $\boldsymbol{\lambda}^{p,0}$ as a $P$-dimensional vector whose $p$-th element is a small positive number $\delta_p$ and all the other elements are 0. The $J^0$ is calculated by setting its element on the $i$-th row and the $j$-th column as: $J_{i,j}^0 = \frac{V_i(\lambda^{j,0}) - V_i(\mathbf{0})}{\delta_j}$.

*Step 3.* Set $\boldsymbol{\lambda}^1 = \boldsymbol{\lambda}^0 - (J^0)^{-1}\boldsymbol{V}(\boldsymbol{\lambda}^0)$. End the search if $\boldsymbol{V}(\boldsymbol{\lambda}^1)$ satisfies the complementary slackness conditions (10); i.e., for each $p = 1, \dots, P$, $V_p(\boldsymbol{\lambda}^1) \le 0$ if $\lambda_p^1 = 0$, and $|V_p(\boldsymbol{\lambda}^1)| < \delta$ if $\lambda_p^1 > 0$. Otherwise set $n = 1$ and go to Step 4.

*Step 4.* Set $(J^n)^{-1} = (J^{n-1})^{-1} + \frac{(\boldsymbol{\lambda}^n - \boldsymbol{\lambda}^{n-1}) - (J^{n-1})^{-1} * (\boldsymbol{V}(\boldsymbol{\lambda}^n) - \boldsymbol{V}(\boldsymbol{\lambda}^{n-1}))}{(\boldsymbol{\lambda}^n - \boldsymbol{\lambda}^{n-1})^T * (J^{n-1})^{-1} * (\boldsymbol{V}(\boldsymbol{\lambda}^n) - \boldsymbol{V}(\boldsymbol{\lambda}^{n-1}))} * (\boldsymbol{\lambda}^n - \boldsymbol{\lambda}^{n-1})^T * (J^{n-1})^{-1}$ and $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{n+1} = \boldsymbol{\lambda}^n - (J^n)^{-1}\boldsymbol{V}(\boldsymbol{\lambda}^n)$. End the search if $\boldsymbol{V}(\boldsymbol{\lambda}^{n+1})$ satisfies the complementary slackness conditions (10). Otherwise go to Step 5.

*Step 5.* Define vector operator $\otimes$ as $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 b_1 \\ a_2 b_2 \\ a_3 b_3 \end{bmatrix}$ for any $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. If the number of negative elements in vector $V(\lambda^{n-1}) \otimes V(\lambda^{n+1})$ is larger than that in $V(\lambda^n) \otimes V(\lambda^{n+1})$, set $\lambda^n = \lambda^{n-1}$. Set $n = n + 1$ and return to Step 4.

## 3. Segment-level MR&R models and solution approaches

This section presents the formulation and solution approaches of the segment-level subproblem that jointly optimizes all the three treatments, i.e. preventive maintenance (chip seal), rehabilitation and reconstruction. While the framework in Section 2 applies to almost all segment level subproblems, to stay focused, we present here only a segment-level formulation that is discrete in time but continuous in the pavement condition (i.e., the roughness index) for an infinite planning horizon. Most of the problem formulation, except for the maintenance model, is similar to the one presented in Lee and

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[8] The Broyden's method is the multivariate version of the secant method; see an introduction of the original Broyden's method in Jorge and Stephen, 2006. One can also show that the relaxed program has a unique optimum given that each segment-level problem has a unique optimal solution (similar to Lemma 2). However, unlike the combined-budget case, Algorithm 2 cannot guarantee the global convergence to the optimum.

Madanat (2014a), and is presented in Section 3.1. Regarding the solution approach, an efficient greedy heuristic is developed together with a conventional dynamic programming algorithm, which serves as the benchmark for examining the solution quality (Section 3.2).

### *3.1. General formulation*

The state variables are $\boldsymbol{q}_k = (q_{kt}|t = 0,1,2,\cdots) = (s_k(t), h_{kt}|t = 0,1,2,\cdots)$, where $s_k(t)$ and $h_{kt}$ are the pavement roughness index and the pavement's age (number of years since the last reconstruction), respectively, for segment $k$ in year $t$. The decision variables are $\boldsymbol{x}_k = (v_{kt}, \omega_{kt}, x_{kt,1}, x_{kt,2}, x_{kt,3}|t = 0,1,2,\cdots)$, where the binary variable $x_{ktp}$ ($p = 1,2,3$) is equal to 1 if a maintenance (corresponding to $p = 1$), rehabilitation ($p = 2$) or reconstruction ($p = 3$) activity is executed in year $t$ for segment $k$, respectively, and 0 otherwise; $v_{kt}$ and $\omega_{kt}$ represent the maintenance and rehabilitation intensities in year $t$ for segment $k$, respectively. The full formulation is presented as follows:

$$\min \ Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) + \textstyle\sum_{p=1}^{3} C_{kp}(\boldsymbol{q}_k, \boldsymbol{x}_k) \tag{11a}$$

$$\text{subject to: } C_k^U(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} \int_t^{t+1} l_k\left(c_k^1 s_k(u) + c_k^2\right) e^{-ru} du \tag{11b}$$

$$C_{k,1}(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} x_{kt,1} \left(\gamma_k^1 v_{kt} + \gamma_k^2\right) e^{-rt} \tag{11c}$$

$$C_{k,2}(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} x_{kt,2} \left(m_k^1 \omega_{kt} + m_k^2\right) e^{-rt} \tag{11d}$$

$$C_{k,3}(\boldsymbol{q}_k, \boldsymbol{x}_k) = \textstyle\sum_{t=0}^{\infty} x_{kt,3} \left(z_k^1 + z_k^2 l_k\right) e^{-rt} \tag{11e}$$

$$\bar{b}_k - b_{kt} = x_{kt,1} E_k\left(v_{kt}, s_k(t)\right), \forall t \tag{11f}$$

$$E_k\left(v_{kt}, s_k(t)\right) = \frac{\alpha_k v_{kt}}{\left(s_k(t)\right)^{\beta_k}} \tag{11g}$$

$$0 \le v_{kt} \le D_{kt} = \min\left\{\bar{v}_k, \frac{(\bar{b}_k - b_k^*)\left(s_k(t)\right)^{\beta_k}}{\alpha_k}\right\}, \forall t \tag{11h}$$

$$s_k(t) - s_k^+(t) = x_{kt,2} G_k\left(\omega_{kt}, s_k(t)\right) + x_{kt,3}\left(s_k(t) - s_k^{new}\right), \forall t \tag{11i}$$

$$G_k\left(\omega_{kt}, s_k(t)\right) = \frac{g_k^1 s_k(t)}{g_k^2 s_k(t) + g_k^3} \omega_{kt} \tag{11j}$$

$$0 \le \omega_{kt} \le R_{kt} = \left(\frac{g_k^2}{g_k^1} + \frac{g_k^3}{g_k^1 s_k(t)}\right) \max\left(0, \min\left\{s_k(t) - s_k^*, g_k^1 s_k(t)\right\}\right), \forall t \tag{11k}$$

$$s_k(u) = F_k(s_k^+(t), u - t, h_{kt}^+, b_{kt}), \forall u \in (t, t+1], \forall t \tag{11l}$$

$$F_k(s_k^+(t), u - t, h_{kt}^+, b_{kt}) = s_k^+(t) e^{b_{kt}(u-t)} + f_k l_k (u - t) e^{b_{kt}(h_{kt}^+ + u - t)} \tag{11m}$$

$$\textstyle\sum_{p=1}^{3} x_{ktp} \le 1, \forall t \tag{11n}$$

$$h_{kt}^+ = h_{kt}\left(1 - x_{kt,3}\right), \forall t \tag{11o}$$

$$s_k^{new} \le s_k(t) \le s_k^{max}, \forall t \tag{11p}$$

$$T_k^{min} x_{kt,3} \le h_{kt} x_{kt,3} \le T_k^{max} x_{kt,3}, \ \forall t \tag{11q}$$

$$q_{k0} = (s_k(0), h_{k0}) \tag{11r}$$

The models for the user cost $C_k^U$, maintenance cost $C_{k,1}$, rehabilitation cost $C_{k,2}$ and reconstruction cost $C_{k,3}$ are described in (11b-e), respectively, where $l_k$ is the annual traffic loading on segment $k$; and $c_k^1$, $c_k^2$, $\gamma_k^1$, $\gamma_k^2$, $m_k^1$, $m_k^2$, $z_k^1$ and $z_k^2$ are (non-negative) cost coefficients.

Of note is that the maintenance cost model (11c) is for chip seal only, which is one of the most commonly used preventive maintenance activities (Labi and Sinha, 2003). Here the maintenance intensity variable $v_{kt}$ is defined as the average least dimension (ALD) of chip seal in year $t$ for segment $k$. The non-negative cost coefficients $\gamma_k^1$ and $\gamma_k^2$ depend upon the oil price, geographical location of the pavement, labor cost, etc. Our maintenance effectiveness model is shown by (11f-g). The model is built upon the following two facts: i) the pavement roughness before and after the chip seal are approximately the same but the deterioration rate diminishes, which is consistent with the findings of Mamlouk and Dosa (2014) among others; and ii) the reduction in deterioration rate is a non-increasing function of the pavement roughness level (see Table 2 and Figures 4-7 of Mamlouk and Dosa, 2014)[10]. The $\bar{b}_k$ and $b_{kt}$ in (11f) are the deterioration rates before and after applying chip seal. The mathematical form of (11g) is selected to fit the real test data of chip seal from Mamlouk and Dosa (2014), where parameters $\alpha_k > 0$, $\beta_k \geq 1$. In addition, there should be a technical upper bound for the ALD, $\bar{v}_k$. Also, the deterioration rate has a lower bound $b_k^*$, at which any additional maintenance has no effect. Thus, the effective maintenance intensity is bounded by $D_{kt}$, which is defined in (11h).

Other parts of the segment-level formulation are borrowed from previous studies, mostly from Ouyang and Madanat (2004; 2006) and Lee and Madanat (2014a). Constraints (11i) indicate the roughness index reduction caused by a rehabilitation or reconstruction activity, where function $G_k$ represents the rehabilitation effectiveness as defined in (11j); $s_k(t)$ and $s_k^+(t)$ denote the roughness indices right before and after the activity, respectively; $s_k^{new}$ is the roughness index immediately after a reconstruction; and $g_k^1$, $g_k^2$, and $g_k^3$ are coefficients. Constraints (11k) stipulate the upper bound, $R_{kt}$, for the rehabilitation intensity, where $s_k^*$ is the best possible roughness level after a rehabilitation. Constraints (11l) indicate how the pavement state is updated at moment $u \in (t, t+1]$, where $F_k$ is a history-dependent deterioration model shown in (11m); and $h_{kt}^+$ is the pavement age after the activity. Constraints (11n) ensure that at most one activity is performed every year. Constraints (11o) reset the pavement age to 0 after a reconstruction. Constraints (11p-q) specify the upper and lower bounds of the roughness level and the pavement's lifecycle length. Constraint (11r)

[10] This means maintenance (e.g. chip seal) is less effective when being applied to a pavement in worse condition. However, a previous maintenance cost and effectiveness model (Gu et al., 2012; Lee and Madanat, 2014a, b) resulted in predictions that were at odds with this simple fact. For example, Lee and Madanat (2014a) observed a complicated, non-monotonic trend between deterioration rate reduction and the pavement's roughness level (see Fig. 4a of that paper). In their results, a larger deterioration rate reduction may occur when the roughness level is high. Numerical analysis has verified that this mistake was corrected by using our maintenance model.

defines the initial pavement state.

### *3.2. Solution method*

We first decompose the infinite-horizon optimization problem (11a-r) into two finite-horizon subproblems (Section 3.2.1).[11] Each subproblem has fewer decision variables and is thus easier to solve. We present in Section 3.2.2 two algorithms to solve the subproblems: a dynamic programming algorithm similar to the one used by Lee and Madanat (2014a) and a greedy heuristic. The heuristic can achieve the same solution accuracy as the dynamic programming approach with only a small fraction of the computation time, as is validated later in Section 4.2.

#### *3.2.1. Problem decomposition*

With the augmented state $q_{kt} = (s_k(t), h_{kt})$, the infinite horizon problem still follows a Markov Decision Process (Li and Madanat, 2002; Lee and Madanat, 2015); i.e., the optimal MR&R decisions from year $t$ onwards (and the future pavement states) depend only on the present state $q_{kt}$. Based on the Principle of Optimality (Bellman, 1957), the optimal roughness trajectory after the first reconstruction enters a periodic steady state, since every reconstruction will reset the pavement to $(s_k^{new}, 0)$. The steady-state solution is thus characterized by a fixed lifecycle duration denoted by $T_k$. The period prior to the first reconstruction is termed as the transient period, which will be optimized separately. Therefore, the objective function (11a) is reformulated as follows:

$$\min \ Z_k(\boldsymbol{q}_k, \boldsymbol{x}_k) = Z_k^T(\boldsymbol{q}_k, \boldsymbol{x}_k) + \frac{e^{-rt_k^T}}{1-e^{-rT_k}} Z_k^S(\boldsymbol{q}_k, \boldsymbol{x}_k) \tag{12}$$

where $Z_k^T$ is the discounted cost for the transient period, and $Z_k^S$ is the cost for one steady-state cycle (with a reconstruction activity at the beginning) discounted to the beginning of the cycle, and $t_k^T$ is the time of the first reconstruction. The $Z_k^T$ and $Z_k^S$ are given by the following equations.

$$Z_k^T(\boldsymbol{q}_k, \boldsymbol{x}_k) = \sum_{t=0}^{t_k^T-1} \Big( \int_t^{t+1} l_k\big(c_k^1 s_k(u) + c_k^2\big) e^{-ru} du + x_{kt,1}\big(\gamma_k^1 v_{kt} + \gamma_k^2\big) e^{-rt} + x_{kt,2}(m_k^1 \omega_{kt} + m_k^2) e^{-rt} \Big) \tag{13}$$

$$Z_k^S(\boldsymbol{q}_k, \boldsymbol{x}_k) = \sum_{\tau=0}^{T_k-1} \Big( \int_\tau^{\tau+1} l_k\big(c_k^1 s_k(u) + c_k^2\big) e^{-ru} du + x_{k\tau,1}\big(\gamma_k^1 v_{k\tau} + \gamma_k^2\big) e^{-r\tau} + x_{k\tau,2}(m_k^1 \omega_{k\tau} + m_k^2) e^{-r\tau} \Big) + z_k^1 + z_k^2 l_k \tag{14}$$

In equation (14) we use $\tau$ to denote the "age" in a steady-state lifecycle (counted from 0 starting from the last reconstruction), and $q_{k0} = (s_k^{new}, 0)$.

Note that $Z_k^S$ is independent of the transient period duration $t_k^T$ and the MR&R schedule during that period. We can thus decompose this problem into two subproblems: the first subproblem

[11] It shall be straightforward that finite-horizon problems can be solved similarly.

for optimizing $\frac{Z_k^S}{1-e^{-rT_k}}$, and the second for optimizing $Z_k$ given the optimal solution of the first one. Further note that Lee and Madanat (2014a) proved $\omega_{kt}$ is either 0 or $R_{kt}$ at optimality. One can easily verify by applying Lee and Madanat's method that the same optimality condition is true for our model. Thus, $\omega_{kt}$ can be eliminated from the list of decision variables. Now the two subproblems are summarized as follows:

*Subproblem 1:* Minimize $\frac{Z_k^S}{1-e^{-rT_k}}$ subject to constraints (11f-r) with decision variables $v_{k\tau}, x_{k\tau,1}, x_{k\tau,2}$ $(\tau = 0,1,2,\dots,T_k - 1)$ and $T_k$.

*Subproblem 2:* Minimize $Z_k = Z_k^T + e^{-rt_k^T}\left(\frac{Z_k^S}{1-e^{-rT_k}}\right)^*$ subject to constraints (11f-r) with decision variables $v_{kt}, x_{kt,1}, x_{kt,2}$ $\left(t = 0,1,2,\dots,t_k^T - 1\right)$ and $t_k^T$, where $\left(\frac{Z_k^S}{1-e^{-rT_k}}\right)^*$ is the optimal value found in subproblem 1.

*3.2.2. Algorithms for the subproblems*

we first use a dynamic programming algorithm modified from the one developed by Lee and Madanat (2014a, 2015). The algorithm is relegated to Appendix D in the interest of brevity. To apply the algorithm, we discretize both the maintenance intensity $v_{k\tau}$ and the pavement roughness level $s_k(\tau)$ into $d+1$ and $N+1$ points, respectively, where $d$ and $N$ are integers. As $d$ and $N$ approach to infinity, the dynamic programming solution converges to the global optimum. Thus, solutions of the dynamic programming approach can be used as benchmarks for verifying the solution quality of a much faster greedy heuristic. We next describe the details of this heuristic algorithm.

The heuristic is based upon the assumption that preventive maintenance is much cheaper than rehabilitation, which is true for most prevailing preventive maintenance treatments including chip seal (Labi and Sinha, 2003). Thus, we start by seeking solutions where maintenance is performed more frequently, while rehabilitation is adopted only when that becomes a must. For the same reason, we also postulate that a maintenance activity is always executed with the maximum intensity $D_{k\tau}$. (This postulation was verified by our extensive numerical tests.) To further avoid solutions with high frequency of rehabilitation, we specify a lower bound of roughness level, $W_k$, below which rehabilitation should not be executed. Different values of $W_k$ were used in the algorithm to balance off the solution quality and the computational efficiency. The algorithm for subproblem 1 is presented as follows:

**Algorithm 3:**

For each $W_k$, do the following and record the least-cost solution:

*Step 1.* Initialize $\tau = 1, cost_2 = \infty$.

*Step 2*. If $\tau < T_k^{max}$, find the action in year $\tau$ from the action set: $\{Do\text{-}nothing\ (x_{k\tau,1} = x_{k\tau,2} = 0),\ Maintenance\ (x_{k\tau,1} = 1, x_{k\tau,2} = 0),\ Rehabilitation\ (x_{k\tau,1} = 0, x_{k\tau,2} = 1)\}$, which minimizes the objective function $\frac{Z_k^S}{1-e^{-rT_k}}$ for the MR&R plan generated by the following steps 2.1-2.3. Record the minimum objective value as $cost_1$:

*Step 2.1*. Keep the recorded MR&R plan before year $\tau$ and execute the selected action in year $\tau$.

*Step 2.2*. For each year $y > \tau$, execute a maintenance with the maximum intensity $D_{ky}$; and if $s_k(y+1) > s_k^{max}$, replace this maintenance in year $y$ by a rehabilitation.

*Step 2.3*. Among year $T_k^{max}$ and all those year of rehabilitation between $T_k^{min}$ and $T_k^{max}$, find the year in which a reconstruction minimizes the objective function $\frac{Z_k^S}{1-e^{-rT_k}}$.

The selected action of year $\tau$ should also satisfy the following conditions: $s_k(\tau) > W_k$ if the selected action is rehabilitation; and $s_k(\tau+1) < s_k^{max}$ if the selected action is executed in year $\tau$.

*Step 3*. If $T_k^{min} \le \tau \le T_k^{max}$, calculate the objective function $\frac{Z_k^S}{1-e^{-rT_k}}$ associated with the following MR&R plan: keep the recorded plan before year $\tau$ and execute reconstruction in year $\tau$. Set $cost_2 = \frac{Z_k^S}{1-e^{-rT_k}}$.

*Step 4*. If $\tau = T_k^{max}$ or $cost_2 < cost_1$, record the reconstruction in year $\tau$, end; otherwise, set $\tau = \tau + 1$ and go to Step 2.

Only minor changes are made to the above algorithm when it is applied to subproblem 2. Particularly, $\tau$ is initialized by 0 instead of 1; the objective function is changed to $Z_k = Z_k^T + e^{-rt_k^T}\left(\frac{Z_k^S}{1-e^{-rT_k}}\right)^*$; and finally, the time range for reconstruction is replaced by $\left[T_k^{min'}, T_k^{max'}\right]$, where $T_k^{min'} = \max\{0, T_k^{min} - h_{k0}\}$ and $T_k^{max'} = \max\{0, T_k^{max} - h_{k0}\}$.

## 4. Numerical case studies

Most of the numerical experiments presented in this section are for a pavement system with 100 heterogeneous segments. Although our approach is able to optimize for pavement systems that are 10 times larger within reasonable computation time (see Section 4.5), we choose this medium-size system for analysis simply because it is easier to run for a large batch of numerical experiments with various parameter values. We are thus able to discuss the general findings and insights unveiled by these results. Section 4.1 describes the parameter values. Section 4.2 examines the solution quality and computation efficiency of the segment-level greedy heuristic. The system-level case studies under the combined and separate budget constraints are discussed in Sections 4.3 and 4.4, respectively. The computational efficiency of our solution method is examined in Section 4.5.

### 4.1. Parameter values

Most parameter values used in our numerical cases are summarized in Table 3. The cost parameters $\gamma_k^1$, $\gamma_k^2$ are derived from the empirical cost model of chip seal in Labi and Sinha (2003); the parameters for the chip seal effectiveness model $(\alpha_k, \beta_k, \bar{v}_k)$ are obtained by fitting the model to the data in Mamlouk and Dosa (2014); the other parameter values are borrowed from Lee and Madanat (2014a). To account for heterogeneous segments, we specify that the initial pavement states, traffic loading, and some cost coefficients follow certain uniform distributions, which are denoted by the form of $U[a, b]$ in the table.

Table 3. Parameter values

| Parameter | Value | Unit | Parameter | Value | Unit |
|---|---|---|---|---|---|
| $c_k^1$ | $U[20500,22500]$ | \$/IRI/km/lane/ million ESAL | $\bar{b}_k$ | 0.04 | - |
| $c_k^2$ | 0 | - | $b_k^*$ | 0.025 | - |
| $m_k^1$ | $U[10000,12000]$ | \$/mm/km/lane | $z_k^1$ | 900000 | \$/km/lane |
| $m_k^2$ | $U[140000, 170000]$ | \$/km/lane | $z_k^2$ | 917000 | \$·year/km/ million ESAL |
| $g_k^1$ | 0.66 | - | $s_k^*$ | 0.8 | IRI |
| $g_k^2$ | 7.15 | mm/IRI | $s_k^{new}$ | 0.75 | IRI |
| $g_k^3$ | 18.3 | mm | $s_k^{max}$ | 6 | IRI |
| $\gamma_k^1$ | 130 | \$/mm/lane/km | $f_k^*$ | 0.093 | IRI·lane·year/ million ESAL |
| $\gamma_k^2$ | 300 | \$/lane/km | $T_k^{min}$ | 20 | year |
| $\alpha_k$ | 0.002 | - | $T_k^{max}$ | 60 | year |
| $\beta_k$ | 1.483 | - | $s_k^0$ | $U[1,3]$ | IRI |
| $\bar{v}_k$ | 14 | mm | $l_k$ | $U[0.4,0.9]$ | million ESAL/year/lane |
| $r$ | 0.07 | - | | | |

### 4.2. Validation of the segment-level greedy heuristic algorithm

To verify the quality of the segment-level greedy heuristic, we test 216 numerical cases with varying values of $\lambda_p$ $(p = 1,2,3)$, $q_{k0}$, $l_k$, and $r$: $\lambda_p \in \{0,4\}$, $q_{k0} = (s_k(0), h_{k0}) \in \{(1,3),(2,8),(4,15)\}$, $l_k \in \{0.5,0.8,1.2\}$, $r \in \{0.05,0.07,0.1\}$. Note that the agency cost of treatment $p$ in the objective function is multiplied by the weight $1 + \lambda_p$. The other parameters take values as in Table 3. The numerical case studies are carried out via Matlab R2014a on a PC with Inter® Xeon® 3.60GHz CPU, 32.0GB RAM, and Windows 10 Pro 64-bit.

We compare the greedy heuristic against two instances of the dynamic programming algorithm: where $N = d = 3$ (denoted as DP1), and where $N = 300,\ d = 5$ (denoted as DP2). The solutions generated from DP2 is treated as the global optima because no meaningful improvement of the solutions was observed by further increasing $N$ or $d$. The runtimes and the cost gaps of the heuristic and the DP algorithms are summarized in Table 4, where the cost gaps are defined as:

$$\frac{\text{cost of the greedy heuristic or DP1} - \text{cost of DP2}}{\text{cost of DP2}}$$

Both the averages and the maxima of all the 216 cases are presented.

Table 4. Runtimes and cost gaps for the greedy heuristic and the dynamic programming algorithms

| | Greedy heuristic | DP1 | DP2 |
|---|---|---|---|
| Average runtime (second) | 1.20 | 49.21 | 1439.31 |
| Maximum runtime (second) | 1.47 | 73.43 | 1981.32 |
| Average total cost gap | 0.37% | 0.41% | - |
| Maximum total cost gap | 3.56% | 2.05% | - |
| Average maintenance cost gap | 0.29% | 0.28% | - |
| Maximum maintenance cost gap | 4.17% | 3.83% | - |
| Average rehabilitation cost gap | 0.36% | 0.52% | - |
| Maximum rehabilitation cost gap | 4.35% | 2.69% | - |
| Average reconstruction cost gap | 0.42% | 0.40% | - |
| Maximum reconstruction cost gap | 3.98% | 2.46% | - |

The tabulated values confirm that our heuristic algorithm produces solutions that are very close to the global optima. Note that the average gap in the total cost is only 0.37%. Comparison between the greedy heuristic and DP1 shows that both algorithms furnished solutions of similar quality, but our heuristic took much shorter (about 97% less) runtimes. We will thus use the greedy heuristic in the following sections to ensure the system-level optimization is solved in reasonable runtimes. Recall that our system-level approach preserves the solution quality as long as the segment-level subproblems are solved near the optimality.

### *4.3. Under the combined budget constraint*

First, we randomly generate a 100-segment pavement system, and optimize the total discounted cost for a range of combined annual budget: $B \in [4 \times 10^6, 5 \times 10^6]$ \$/year. The optimal total discounted cost and the cost components are plotted against $B$ as the solid curves in Fig. 1. These curves start from $B = 4.02 \times 10^6$ \$/year on the left because this value of $B$ represents the minimum budget required to find a feasible MR&R plan. This minimum required budget can be calculated by optimizing the decomposed problems (4a-c) with a sufficiently large $\lambda$. The figure shows that the optimal total cost (the solid curve with dot markers) decreases as $B$ grows, until it reaches a threshold of $4.61 \times 10^6$ \$/year as marked by the arrow. This threshold represents the maximum budget needed for the pavement system; i.e., any additional budget would be redundant, and the optimal total cost would stay the same ($11.73 \times 10^7$\$).

The figure also shows that the user cost (the triangle-marked solid curve) also decreases as $B$ increases, which is as expected. Meanwhile, the rehabilitation cost (the "x"-marked solid curve) generally diminishes, while the reconstruction cost (the square-marked solid curve) increases with $B$. This means with more budget to spend, the agency should apply more reconstruction but less rehabilitation to reduce the user cost. On the other hand, when the budget is highly limited, more rehabilitation activities should be performed to extend the pavements' service life. The maintenance cost (the diamond-marked curve near the bottom of the figure) is much lower than the other cost components, and is insensitive to $B$. This is because there is no incentive to trade off the maintenance activities: they are very cheap, but have considerable effects on the pavements.

To examine how adding maintenance affects the optimal MR&R plan, we compare the above total cost and cost components against those for the optimal R&R plans (i.e., no maintenance). The R&R costs are plotted as the dashed curves in Fig. 1. Comparison reveals a total cost saving of 6.3~7.5% from applying maintenance for $B \in [4.43 \times 10^6, 5 \times 10^6]$ \$/year. The minimum annual budget required is also reduced by 9.3% (from $4.43$ to $4.02 \times 10^6$ \$/year). Comparison between the cost components reveals that adding maintenance usually results in a lower reconstruction cost but a higher rehabilitation cost. This means maintenance extends the pavements' service life, which in turn entails more rehabilitation activities.

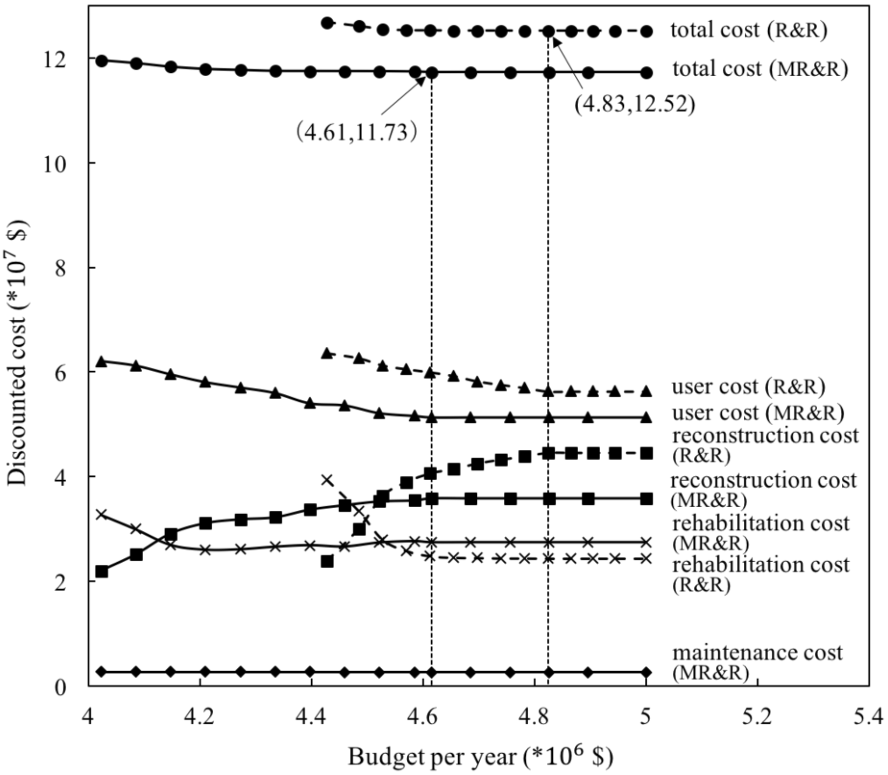
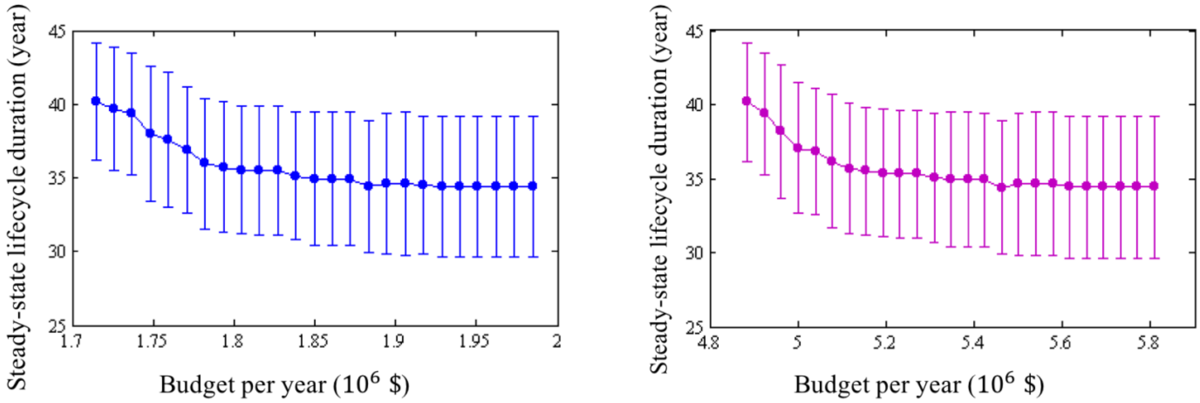



**Fig. 1.** Effects of the combined agency budget on the system-level optimal costs

One may wonder how the initial pavement conditions may affect the optimal MR&R plans for individual segments. Here we plot against the budget constraint the distributions of i) steady-state lifecycle duration (Fig. 2a and b) and ii) number of rehabilitations per steady-state lifecycle (Fig. 3a and b) of the 100 segments. Fig. 2a and 3a are for a system with good initial conditions ($s_k(0)\sim U[0,1], \forall k$), and Fig. 2b and 3b are for the same system but with poor initial conditions ($s_k(0)\sim U[2,3], \forall k$). In each figure, a black dot indicates the mean value (lifecycle duration or rehabilitation count) of all the pavement segments for a specific $B$, and the associated error bar describes the range of two standard deviations centered at the mean. As expected, both the mean lifecycle duration and the mean rehabilitation count decrease as $B$ increases until the constraint becomes unbinding, which is consistent with the findings from Fig. 1. Smaller standard deviations are observed for smaller $B$, indicating that a tighter budget tends to "homogenize" the segment-level MR&R plans.

Comparison between Fig. 2a and b unveils that the mean and standard deviation of lifecycle durations vary along very similar paths as $B$ increases, despite the largely different initial pavement conditions. A high similarity is also observed between Fig. 3a and b for the distribution of rehabilitation counts per lifecycle. This means the steady-state MR&R plans of individual segments are *almost* independent on the initial conditions of those segments. Scrutinization of the numerical results shows that most of the pavement segments have nearly (but not exactly) the same steady-state

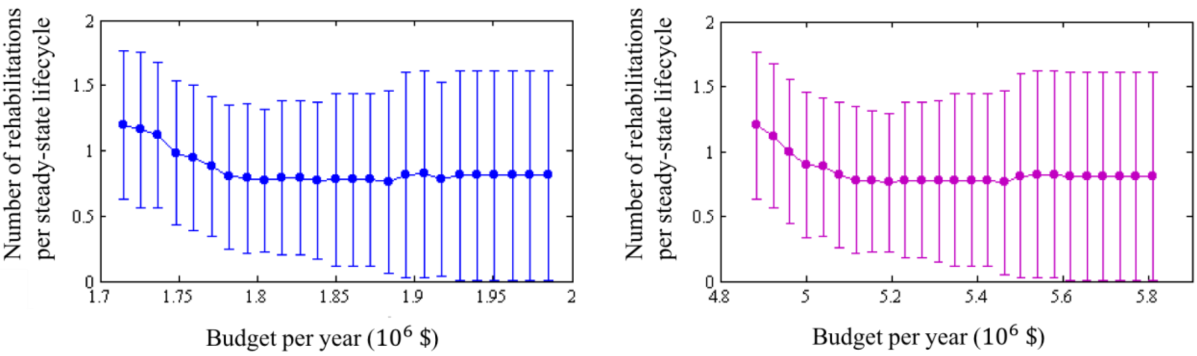
MR&R plans between the two cases.




(a) (b)

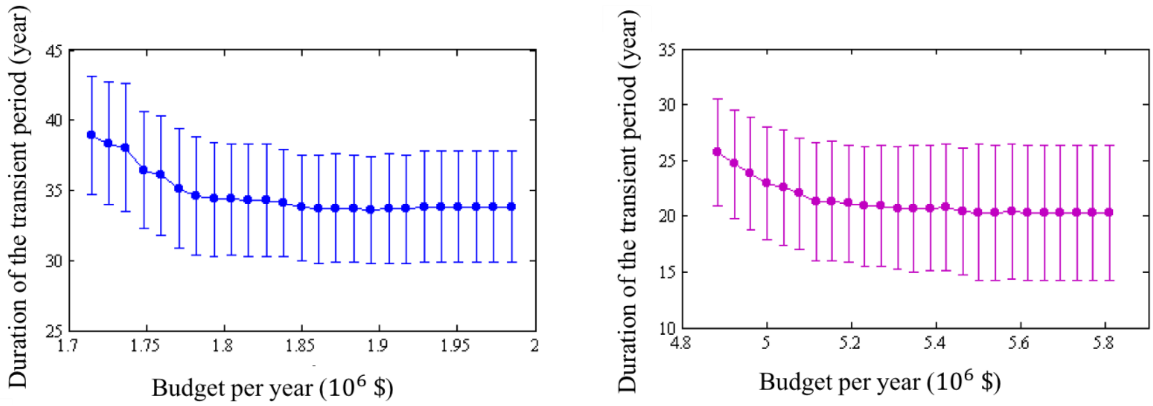
**Fig. 2.** Distribution of steady-state lifecycle durations versus the budget constraint: (a) the case with good initial conditions; (b) the case with poor initial conditions

On the other hand, the initial pavement conditions do have a significant effect on the optimal MR&R plans for each segment's transient period; see the large differences between the distributions of the transient period durations (Fig. 4a and b), and between the distributions of the rehabilitation counts in the transient periods (Fig. 5a and b), for the cases with good and poor initial conditions. Worse initial conditions entail earlier first reconstruction and more rehabilitation activities during the transient periods. Note that the same findings have been observed for other instances of pavement systems of various sizes.



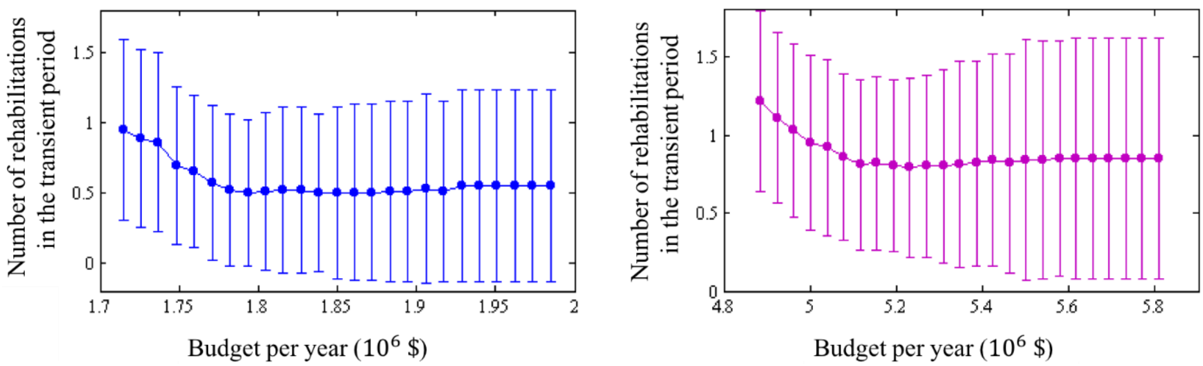

(a) (b)

**Fig. 3.** Distribution of rehabilitation counts per steady-state lifecycle versus the budget constraint: (a) good initial conditions; (b) poor initial conditions




(a) (b)

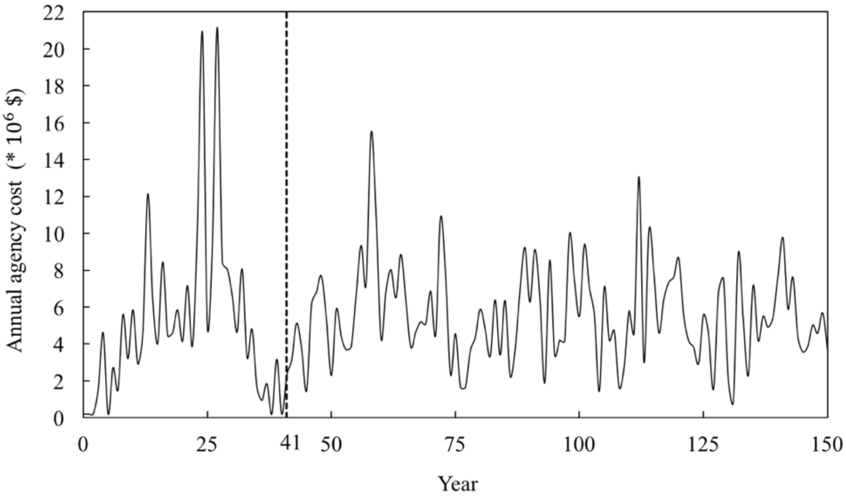
**Fig. 4.** Distribution of the first lifecycle's duration versus the budget constraint: (a) good initial conditions; (b) poor initial conditions



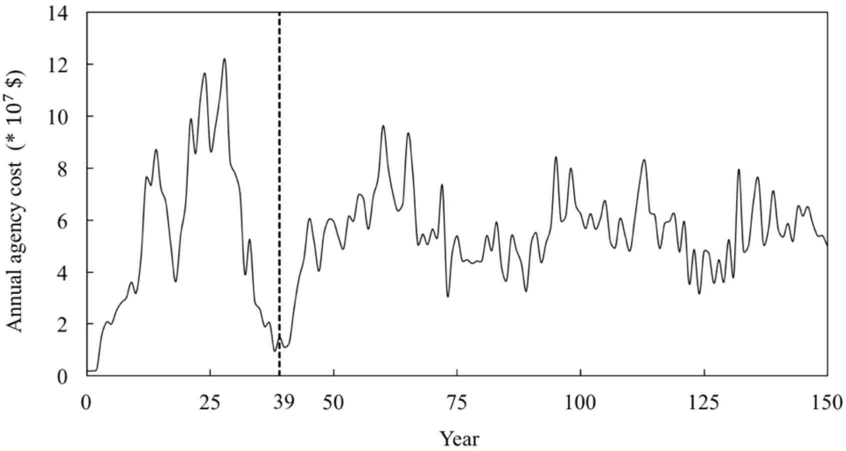

(a) (b)

**Fig. 5.** Distribution of rehabilitation counts in the first lifecycle versus the budget constraint: (a) good initial conditions; (b) poor initial conditions

One may also wonder what the optimal MR&R plan and the minimum total cost would be if the annual budget cannot be transferred across the years. Although solving a problem with budget transfers prohibited is out of the scope of this paper, we can get a rough idea by looking at the actual annual expenditures for our optimal MR&R plan, in which budget transfers across years are allowed. We plot the actual annual agency cost from year 1 to year 150 under the optimal MR&R plan for a 100-segment system with $B = 4.42 \times 10^6$ \$/year (Fig. 6a) and a 1000-segment system with $B = 4.38 \times 10^7$ \$/year (Fig. 6b). Both figures show large variations in the annual agency expenditures. The variation is especially large for a smaller-sized pavement system, and during the transient period of the pavement system (note the much larger variation before the dashed vertical line in both figures, which marks the time when the last segment enters a steady state). This implies that, if a constant, non-transferable budget is set in each year, the resulting MR&R plan would be suboptimal, and the optimal cost would likely be much larger than what we obtain in this paper. To optimally utilize the budget, the agency should always seek to borrow and lend money over the years (e.g., via some financial tools).




(a)




(b)

**Fig. 6**. Annual agency costs under optimal MR&R plans with budget transfers allowed: (a) a system of 100 segments; (b) a system of 1000 segments

### *4.4. Under the separate budget constraints*

In reality, an agency often manages separate budgets for different treatments, as discussed in Lee and Madanat (2015). In this section we revisit how this suboptimal practice affects the performance of the optimal MR&R plan. We examine the same 100-segment pavement system analyzed in Fig. 1, but now under the separate budget constraints. For the clarity of illustration, we here present the results of a reduced problem with two budget constraints only: one for reconstruction and the other for maintenance and rehabilitation combined.[12] Fig. 7a plots a contour map of the optimal total cost for

[12] We choose to present the results of this reduced problem simply for the sake of clarity. Note now the effects of the two budget constraints can be clearly illustrated by two-dimensional contour maps (like Fig. 7a-d). A three-budget-constraint problem can also be solved by our approach, but the effects of the three budget constraints cannot be presented in a similar way in the paper. The analysis of the reduced problem does not

the annual reconstruction budget ranging in $[0, 5.5 \times 10^6]$ \$/year, and the annual maintenance and rehabilitation budget in $[0, 5 \times 10^6]$ \$/year. Each thin, solid curve in the figure represents a contour line with the total discounted cost marked on the curve (in the unit of $10^7$ \$). Examination of this figure unveils interesting findings that complement those in the literature.

First of all, no contour line is present in the region in the lower-left part of Fig. 7a (labeled "***INF***"), because the MR&R optimization problem is infeasible in this region due to insufficient budgets. Note that the area on the left side of the vertical dashed line at $0.36 \times 10^6$ \$/year (the minimum reconstruction budget associated with $T_k^{max}$) all belongs to region ***INF***, regardless of the maintenance and rehabilitation budget. On the other hand, no contour line exists in the rectangular region in the upper-right corner of Fig. 7a (labeled "***A***"), because in this region both budget constraints are unbinding and the optimal total cost remains constant at $11.73 \times 10^7$ \$. Note the bottom-left corner of region ***A*** indicates the maximum budgets needed: $2.51 \times 10^6$ \$/year for reconstruction and $2.10 \times 10^6$ \$/year for maintenance and rehabilitation combined.

The remaining part of the figure is divided into three regions: ***B***, ***C***, and ***D***, as demarcated by the thick solid lines in the figure. Region ***B*** refers to the set of cases where the reconstruction budget constraint is unbinding and the maintenance and rehabilitation budget constraint is binding. Hence the contours in this region are horizontal lines. Region ***C***, on the other hand, is where the maintenance and rehabilitation budget constraint is unbinding but the reconstruction one is binding. Finally, region ***D*** is where both budget constraints are binding. Note that each unbroken contour line is tangent to a line with slope -1, and the tangent point indicates the optimal solution under the combined budget constraint. Some of these combined-budget-constraint problem solutions are shown as black dots on the contour lines of 11.90, 11.80, and 11.75$\times 10^7$ \$. The lower boundary of ***D*** is also tangent to a line with slope -1 (the dashed line shown in Fig. 7a); this dash line specifies the minimum budget required for the combined budget scenario ($4.02 \times 10^6$), which is consistent with Fig. 1. This is also intuitive: if a feasible MR&R plan is found for a given pair of separate budget constraints, then the corresponding problem when all the budgets are combined is also feasible.

Fig. 7b shows the contour map of the percentage of cost savings by comparing the combined-budget-constraint scenario against the separate-budget-constraint one. The figure shows a cost saving of up to 4% when the reconstruction budget is small. On the other hand, if only the maintenance and rehabilitation budget is small, the cost saving is below 2%. The dashed line with slope -1 indicates the maximum required combined budget, and the contour lines above the dashed line should overlap with the contours of the optimal total cost.

To further illustrate the effectiveness of maintenance, Fig. 7c compares the five solution

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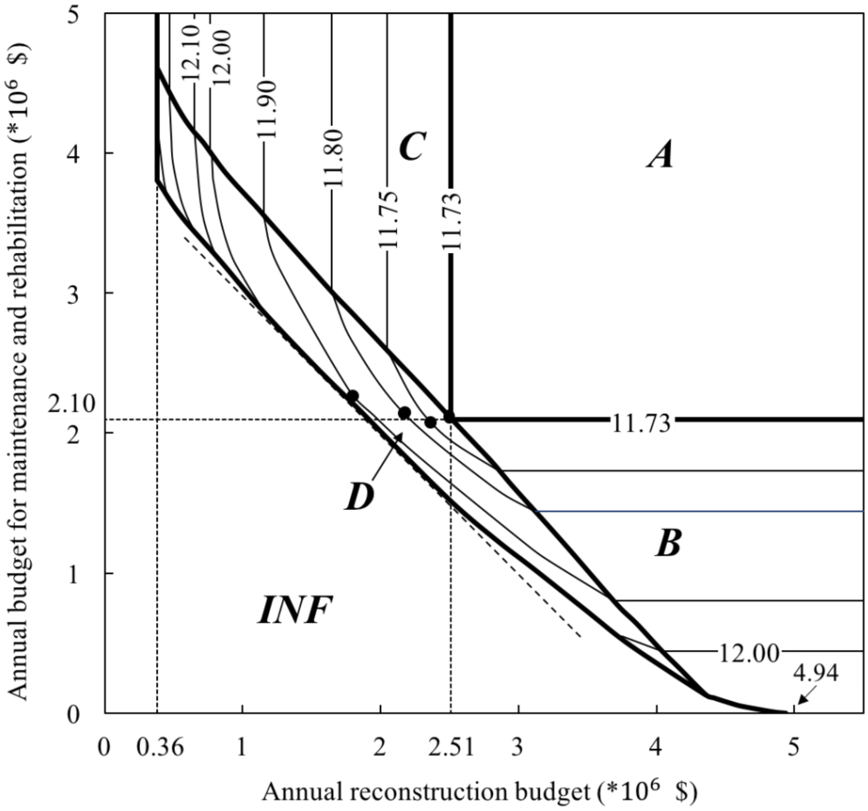
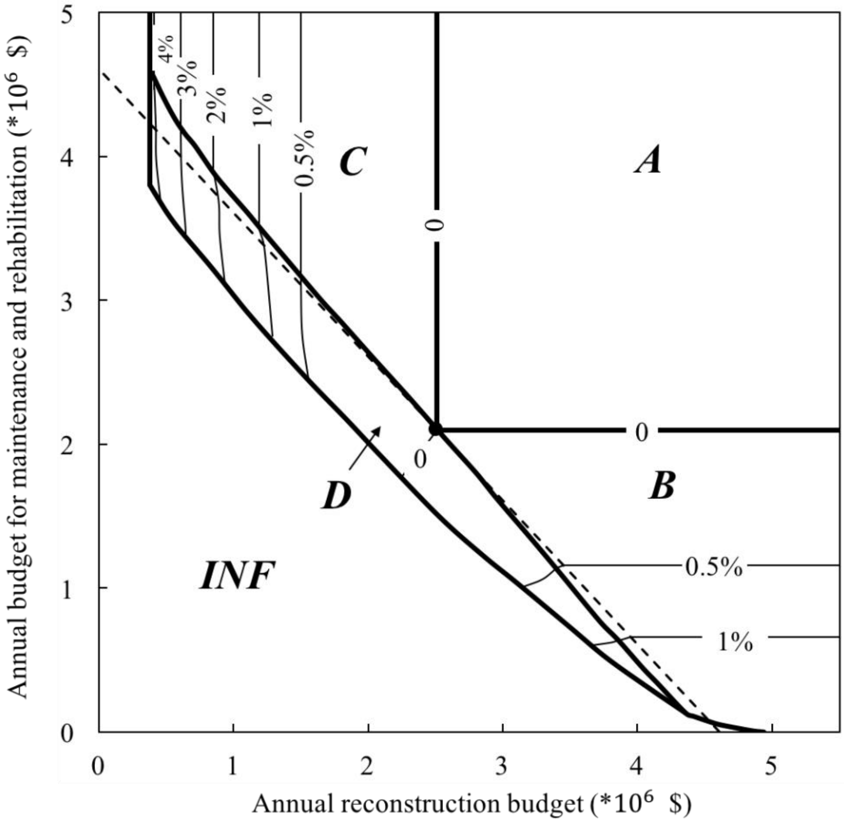
compromise our findings since the maintenance cost is always small and easy to accommodate; see again Fig. 1.

regions defined above (***A***, ***B***, ***C***, ***D***, and ***INF***) against the regions for the optimal R&R plan (i.e. without preventive maintenance). The solution regions for the R&R case are demarcated by thick, dashed lines in the figure. The figure shows that when preventive maintenance is included, region ***D*** expands and moves downward, while region ***INF*** diminishes. This means including maintenance can largely reduce the budget needed to keep the pavements workable.

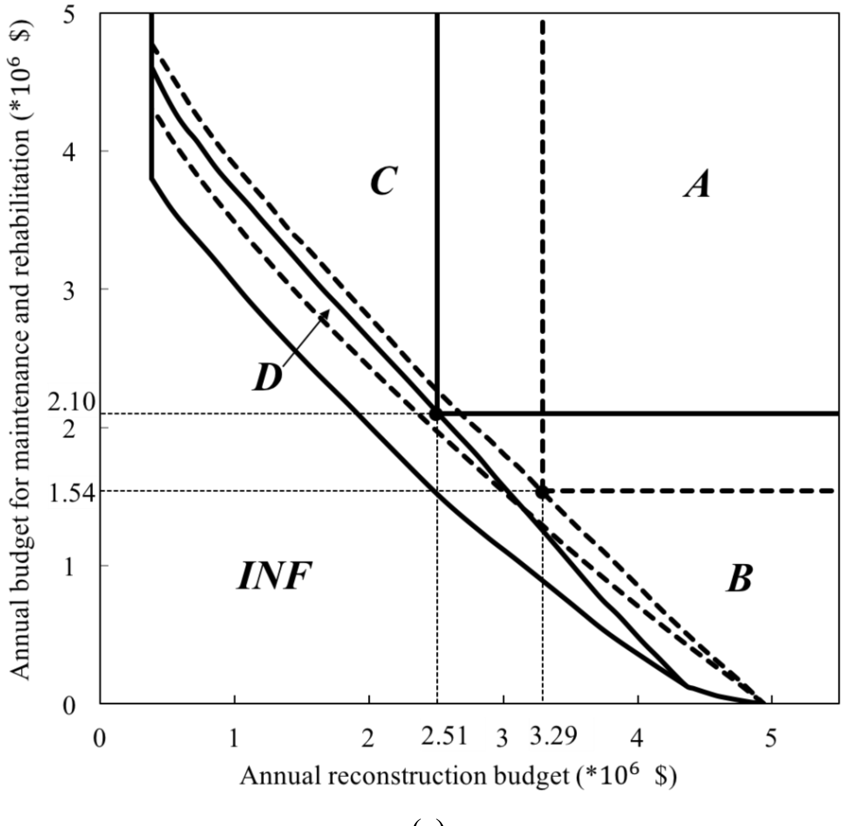



(a)



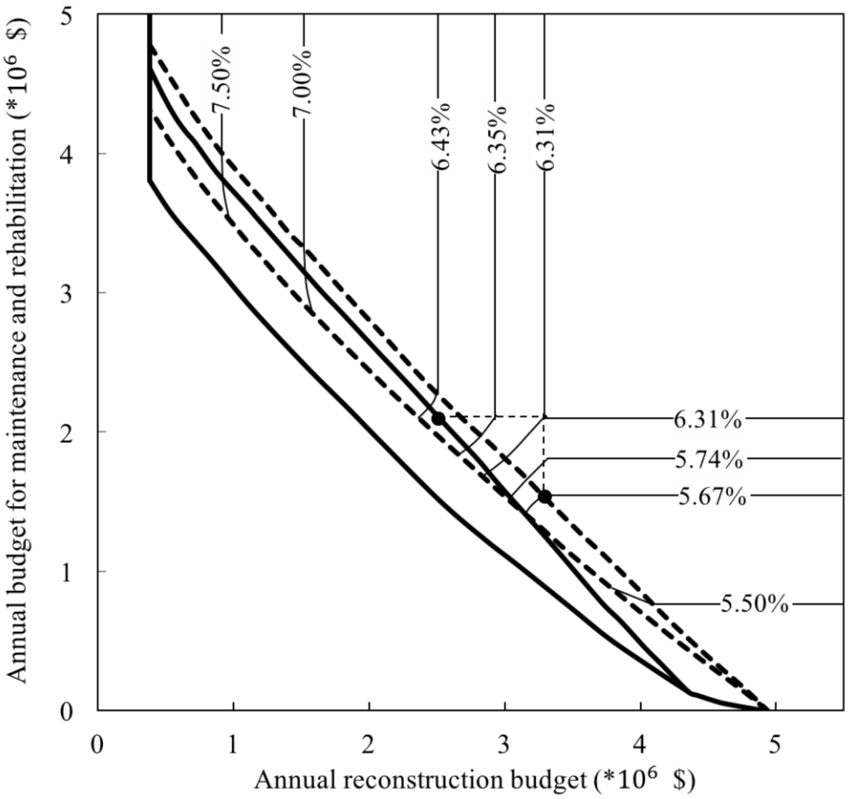
Annual budget for maintenance and rehabilitation (*10^6 $)
Annual reconstruction budget (*10^6 $)
0
1
2
3
4
5
4%
3%
2%
1%
0.5%
C
A
B
D
INF
0
0.5%
1%


(b)



Annual budget for maintenance and rehabilitation (*10^6 $)
Annual reconstruction budget (*10^6 $)
2.10
1.54
2.51
3.29
C
A
B
D
INF


(c)




(d)

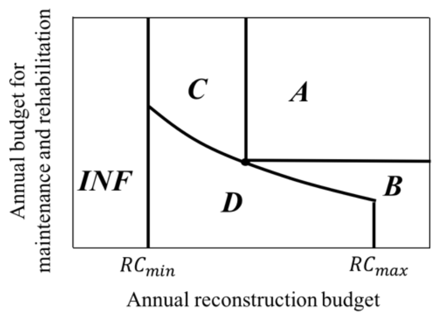
**Fig. 7.** Results of the case under separate budget constraints: (a) contours of optimal total cost and the solution regions; (b) percentage of cost savings from optimally allocating the budget for different treatments; (c) comparison of the solution regions with and without maintenance; (d) percentage of cost savings from adding maintenance.

Finally, the percentage of cost saving between MR&R and R&R is plotted in Fig. 7d. It shows that including maintenance can bring an over 5% reduction in the optimal total cost for most of the cases. Highest cost savings (almost 8%) are achieved when the reconstruction budget is small, because maintenance can extend the pavements' lifecycles and thus reduce the need for reconstruction.

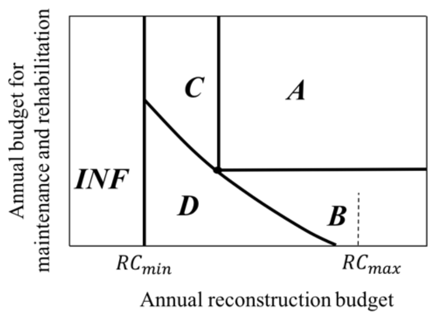
The solution regions shown in Fig. 7c are different from those presented by Lee and Madanat (2015). Specifically, in Lee and Madanat the right boundary of region ***INF*** is a vertical line, and the lower boundary of region ***D*** is the horizontal axis; see Fig. 4 in their paper. The difference is due to the different input parameters used in our case studies. In general, there are seven patterns of the solution regions that may arise from real-life pavement systems, which are illustrated in Fig. 8a-g.

The result in Lee and Madanat belongs to the pattern shown by Fig. 8a, where $RC_{min}$ and $RC_{max}$ denote the minimum and maximum reconstruction costs that are required when the lifecycle duration is $T_k^{max}$ and $T_k^{min}$, respectively. (Note that this is the only pattern described in Lee and Madanat, 2015.) This pattern occurs if: i) a feasible solution exists when no maintenance or rehabilitation is applied, and only the minimum reconstruction is executed; and ii) the maximum reconstruction budget $RC_{max}$ will be binding when no maintenance or rehabilitation is applied. If only condition ii) is false, i.e., $RC_{max}$ is unbinding even if no maintenance or rehabilitation is applied, then the upper boundary of region $\boldsymbol{D}$ would hit the horizontal axis before crossing the vertical line at $RC_{max}$. This will render the pattern shown by Fig. 8b.

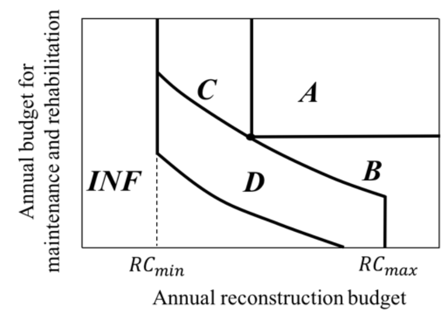
On the other hand, if the above condition i) is false, then the lower boundary of region $\boldsymbol{D}$ will decline as the reconstruction budget increases. This oblique lower boundary may end by: I) hitting the horizontal axis (before reaching $RC_{max}$); II) crossing the upper boundary of $\boldsymbol{D}$ (before reaching $RC_{max}$); and III) crossing the vertical line at $RC_{max}$. Case I) can be further divided into two patterns: when the upper boundary of $\boldsymbol{D}$ ends at the vertical line at $RC_{max}$ (Fig. 8c), and when that boundary also ends at the horizontal axis (Fig. 8d). In case II), the two boundaries of region $\boldsymbol{D}$ merge to a single line which is decreasing as reconstruction budget increases. This line will cross the horizontal axis (Fig. 8e) or the vertical line at $RC_{max}$ (Fig. 8f). Finally, case III) will render the patterns described by Fig. 8g. Note any interface that appears on the right of $RC_{max}$ has to be horizontal. The results shown in Fig. 7a-d belong to the pattern in Fig. 8e.



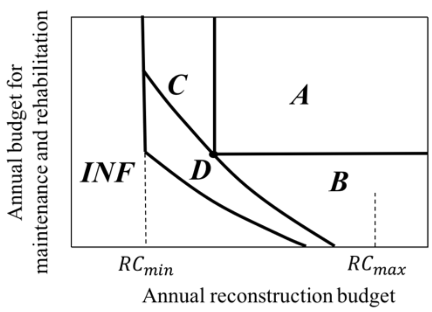

(a)




(b)




(c)




(d)

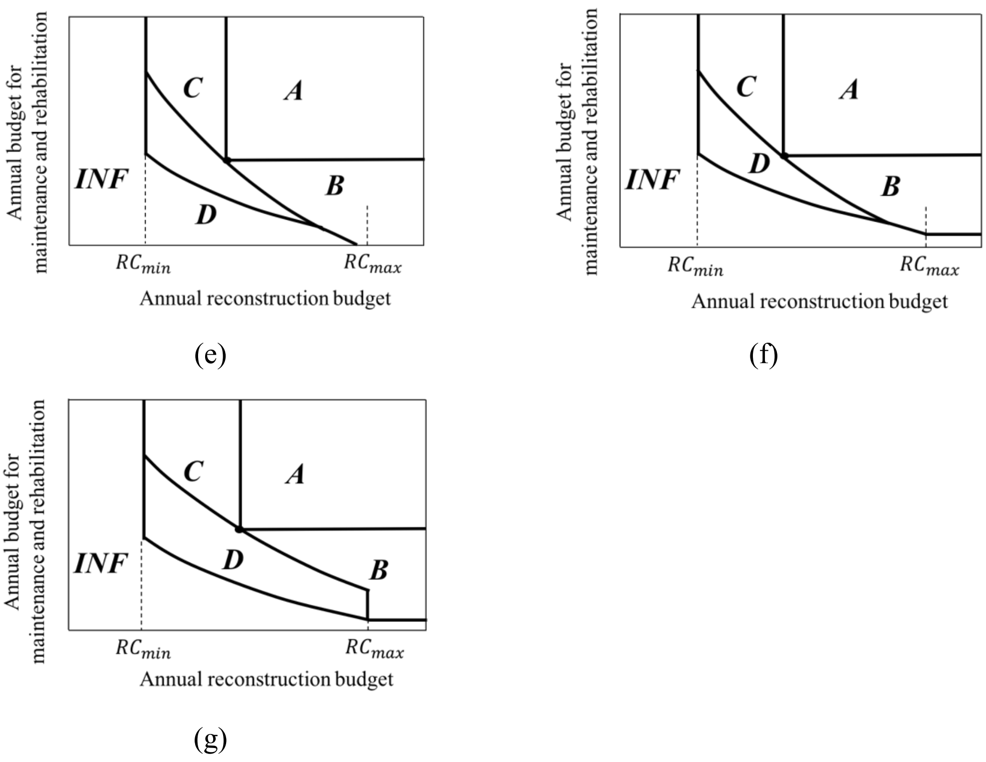



**Fig. 8.** Patterns for the solution regions ***A***, ***B***, ***C***, ***D***, and ***INF***

### *4.5. Computational efficiency*

The solid dots in Fig. 9 present the computation times of 110 randomly generated numerical instances under the combined budget constraint against the number of pavement segments (ranging 50 to 1000). They were carried out via Matlab R2014a on a PC with Inter® Xeon® 3.60GHz CPU, 32.0GB RAM, and Windows 10 Pro 64-bit. These dots exhibit a clear linear relationship between the computational time and the size of the problem. Similar linear relationship is found for the cases with separate budget constraints. This is because the number of iterations needed for the Lagrange multiplier(s) $\lambda$ (or $\lambda_p$) to converge is uncorrelated with the size of the system. With our selected error tolerance level (1% of the budget), this number of iterations is usually 4-5 under the combined budget constraint, and 20-28 under three separate budget constraints. Note too that a 1000-segment system takes about 1.5 hours to solve under the combined budget constraint, and about 8-10 hours under three separate budget constraints. The runtime is very reasonable for real-world implementation.

In comparison, the GA algorithm developed by Lee and Madanat (2015) for solving the joint R&R optimization (i.e. without maintenance) seems to exhibit a polynomial complexity (see Fig. 6 of the cited work); i.e. the computation time increases much faster than the linear trend. Thus our approach is more computationally efficient than the GA algorithm especially for larger-scale systems.

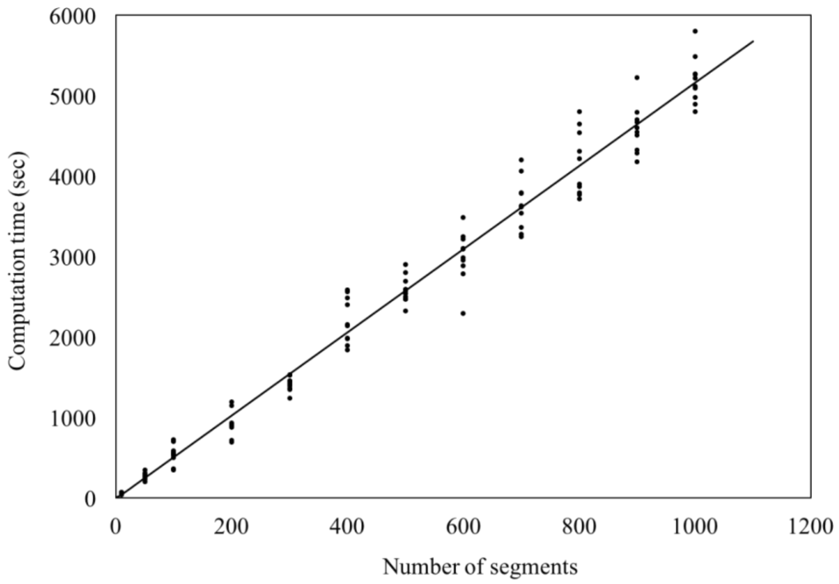



**Fig. 9.** Computation times for the numerical instances under the combined budget constraint

## 5. Conclusions

We formulate a general mathematical model for the joint optimization of MR&R planning for a system of heterogeneous pavement segments under budget constraints. We propose a Lagrange multiplier approach combined with derivative-free quasi-Newton methods to solve the system-level program. The approach relaxes the budget constraints and decomposes the system-level problem into multiple segment-level subproblems, whose solutions can be more easily derived. Hence, our approach can be applied to segment-level models that take any specific forms.

Our work has extended the literature in the realm of pavement MR&R optimization in multiple aspects. We are, to our best knowledge, the first to formulate and solve a full version of system-level MR&R optimization problem that incorporates preventive maintenance activities, which are modeled by a more realistic formulation fitted on the real data. The inclusion of maintenance adds another dimension to the solution space, as compared to the previous system-level studies (e.g. Lee and Madanat, 2015). Despite the added complexity, however, the problem is solved within only moderate runtimes, thanks in part to the derivative-free quasi-Newton methods used to search for the $\lambda'$s, and in part to the efficient segment-level heuristic. More importantly, the runtime increases linearly with the number of segments in a system, which ensures the applicability of our solution approach to large-scale systems. Further, note that the computational efficiency is achieved without compromising the solution quality. Particularly, for the problem under the combined budget constraint, our approach guarantees the global optimality or near-optimality at the system level as long as the segment-level subproblems are solved at or near the optima. High-quality solutions are always preferred because even one additional percent of reduction in the total cost would mean a saving of millions of dollars.

Our numerical case studies reveal a number of useful findings. For example, the results show that by optimally allocating a combined agency budget among the treatments, the minimum total cost can be reduced by up to 4% (see again Fig. 7b). Incorporating maintenance in the optimal MR&R planning will result in a total cost saving of over 6% (see Fig. 7d), and more importantly, it can significantly lower the minimum budget required to keep the pavement system workable (by over 9% in our numerical case; see Fig. 1). Highway agencies can obtain the optimal allocation of the budget for each treatment, and the minimum total budget required from our model. These types of information are very useful for them to prepare for future budget proposals. Managerial insights are also unveiled, including: i) that the agency should perform fewer reconstructions but more rehabilitations when the budget is more limited; ii) that incorporating maintenance will reduce the need for reconstruction but not for rehabilitation (actually the rehabilitation frequency would increase) in the optimal MR&R plan; and iii) that the pavements' initial conditions have a significant effect on the optimal MR&R plans during the transient periods and the minimum total budget required, but have almost no effect on the optimal steady-state MR&R plans. These insights are helpful for agencies to plan for future pavement management activities.

To be sure, our work still has several limitations. For example, some findings and insights summarized from the numerical case studies might be dependent on the specific parameter values we used. In particular, we find the maximum allowable roughness index, $s_k^{max}$ (which defines the worst acceptable condition for a workable pavement), has a significant impact on the cost savings stemmed from optimally allocating the treatment budgets and from incorporating maintenance. Larger $s_k^{max}$ (i.e. higher tolerance for poor pavements) would result in more savings.

The present segment-level models are also limited in that: i) the cost and effectiveness models for a variety of other preventive maintenance treatments (e.g. fog seal and microsurfacing) are not included; ii) the present models for chip seal and rehabilitation fail to account for the influence of a number of factors including the environmental conditions; iii) the roughness index is not a perfect indicator of pavement conditions; and iv) the present user cost model and the assumption of constant traffic loading are also strong simplifications of the reality. However, our system-level approach can still be applied to the more complicated scenarios that address the above practical concerns, should more realistic segment-level models be made available. Work in this regard is underway.

Potential extensions of our work also include: modeling and solving the problem with annual budget constraints that are not transferable across the years; modeling the uncertainties in the deterioration process and MR&R effectiveness; and accounting for other operational and practical constraints like the greenhouse gas emissions, network connectivity, etc. These extensions would require not only a revised formulation of the problem, but also more efficient search algorithms to ensure convergence within reasonable runtimes. Some of these extensions are currently under investigation too.

**Acknowledgements**

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**Appendix A. Sketch of proof of Lemma 1**

First note that the case of $\lambda^* = 0$ is trivial. In the following proof we assume that $\lambda^* \neq 0$ and $\lambda^H \neq 0$ (note it is unlikely that $\lambda^* \neq 0$ and $\lambda^H = 0$ when $\delta_1$ and $\delta_2$ are both small). So from (6a-b), we have:

$$\sum_{k=1}^{K} C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) = \sum_{k=1}^{K} C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right) = B \tag{A1}$$

Then,

$$\left|\sum_{k=1}^{K}\left(C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) - C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right)\right)\right| = \left|\sum_{k=1}^{K}\left(C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) - C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right)\right| \leq$$

$$\sum_{k=1}^{K}\left|C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) - C_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq K \cdot \delta_1 \tag{A2}$$

On the other hand, since $\boldsymbol{x}_k^*(\lambda)$ minimizes $Z_k(\boldsymbol{x}_k) + \lambda C_k(\boldsymbol{x}_k)$, we have:

$$Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) + \lambda^* \cdot C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) \leq Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) + \lambda^* \cdot C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) \tag{A3}$$

and,

$$Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) + \lambda^H \cdot C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) \leq Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) + \lambda^H \cdot C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) \tag{A4}$$

The (A3) and (A4) can be combined into:

$$\lambda^* \cdot \left(C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right)\right) \leq Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) - Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) \leq \lambda^H \cdot \left(C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right)\right) \tag{A5}$$

Hence,

$$\left|\sum_{k=1}^{K}\left(Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right)\right)\right| \leq \max\{\lambda^*, \lambda^H\} \cdot \left|\sum_{k=1}^{K}\left(C_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - C_k\left(\boldsymbol{x}_k^*(\lambda^H)\right)\right)\right| \leq$$

$$\max\{\lambda^*, \lambda^H\} \cdot K\delta_1 \tag{A6}$$

Now we have:

$$\left|\sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - \sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq \left|\sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^*(\lambda^*)\right) - \sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right)\right| +$$

$$\left|\sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) - \sum_{k=1}^{K} Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\right| \leq \max\{\lambda^*, \lambda^H\} \cdot K\delta_1 + \sum_{k=1}^{K}\left|Z_k\left(\boldsymbol{x}_k^*(\lambda^H)\right) - \right.$$

$Z_k\left(\boldsymbol{x}_k^H(\lambda^H)\right)\Big| \leq K \cdot (\max\{\lambda^*, \lambda^H\}\, \delta_1 + \delta_2)$ (A7)

Note in a real pavement system that $C_k(\boldsymbol{x}_k^*(\lambda^H)) - C_k(\boldsymbol{x}_k^*(\lambda^*))$ can be either positive or negative for any $k$; and the positive and negative components of the sum $\sum_{k=1}^{K}\left(C_k(\boldsymbol{x}_k^*(\lambda^H)) - C_k(\boldsymbol{x}_k^*(\lambda^*))\right)$ will cancel out. Thus inequality (A2) may be a very weak one, and as a result, (A7) may be weak too. ■

**Appendix B. Proof of Lemma 2**

We prove Lemma 2 by contradiction. Suppose there exists $\lambda_1 > \lambda_2 \geq 0$, such that $V(\lambda_1) \geq V(\lambda_2)$. We denote $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ as the solutions associated with $\lambda_1$ and $\lambda_2$, respectively; i.e., for each $k = 1,2,\ldots,K$, $\boldsymbol{x}_k^1$ is the unique minimizer of $H_k(\boldsymbol{x}_k, \lambda_1) \equiv C_k^U(\boldsymbol{x}_k) + (1+\lambda_1)C_k(\boldsymbol{x}_k)$, and $\boldsymbol{x}_k^2$ is the unique minimizer of $H_k(\boldsymbol{x}_k, \lambda_2) \equiv C_k^U(\boldsymbol{x}_k) + (1+\lambda_2)C_k(x_k)$.

We then have:

$0 > \sum_{k=1}^{K}\left[H_k(\boldsymbol{x}_k^1, \lambda_1) - H_k(\boldsymbol{x}_k^2, \lambda_1)\right]$
$= \sum_{k=1}^{K}\left\{\left[C_k^U(\boldsymbol{x}_k^1) + (1+\lambda_1)C_k(\boldsymbol{x}_k^1)\right] - \left[C_k^U(\boldsymbol{x}_k^2) + (1+\lambda_1)C_k(x_k^2)\right]\right\}$
$= \sum_{k=1}^{K}\left[C_k^U(\boldsymbol{x}_k^1) - C_k^U(\boldsymbol{x}_k^2)\right] + (1+\lambda_1)\left(V(\lambda_1) - V(\lambda_2)\right)$
$\geq \sum_{k=1}^{K}\left[C_k^U(\boldsymbol{x}_k^1) - C_k^U(\boldsymbol{x}_k^2)\right] + (1+\lambda_2)\left(V(\lambda_1) - V(\lambda_2)\right)$
$= \sum_{k=1}^{K}\left[H_k(\boldsymbol{x}_k^1, \lambda_2) - H_k(\boldsymbol{x}_k^2, \lambda_2)\right] > 0$
Contradiction! ■

**Appendix C. Sketch of proof of the convergence of Algorithm 1**

We assume that $V(\lambda)$ is continuously differentiable everywhere and the unique root of $V(\lambda)$ is $\lambda^*$ (since $V(\lambda)$ is a decreasing function of $\lambda$). We prove the convergence of Algorithm 1 by contradiction. Suppose the stop criterion cannot be attained as $n$ increases. Initially, we have $\lambda^0 < \lambda^1$ and $V(\lambda^0) > 0$. According to Lemma 2, we have $V(\lambda^1) < V(\lambda^0)$. One of the following two cases will occur.

Case 1: $V(\lambda^n) > 0$ for all $n \geq 1$

In this case $\lambda^{n+1} = \lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})} > \lambda^n$ for all the $n$; i.e., the sequence $\{\lambda^n\}$ is strictly increasing. Thus $\{\lambda^n\}$ should be bounded above, because otherwise $V(\lambda^n)$ would be 0 or negative for sufficiently large $n$. That means $\{\lambda^n\}$ has a supremum: $\tilde{\lambda} = \sup\{\lambda^n\}$. Let $V(\tilde{\lambda}) = \kappa$ as shown in Fig. C1a. We have:

$$\lim_{n\to\infty} V(\lambda^n) = \kappa > 0 \quad \text{(C1)}$$

$$\lim_{n\to\infty} \lambda^n = \tilde{\lambda} \quad \text{(C2)}$$

However,

$$\tilde{\lambda} = \lim_{n\to\infty} \lambda^{n+1} = \lim_{n\to\infty} \left(\lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})}\right) = \tilde{\lambda} - \kappa \cdot V'(\tilde{\lambda}) > \tilde{\lambda} \tag{C3}$$

Contradiction!

Case 2: $V(\lambda^n) < 0$ for some $n \geq 1$.

According to Algorithm 1, we have $V(\lambda^{n-1}) \cdot V(\lambda^n) < 0$ for all $n \geq n'$, where $n' = \min\{n | V(\lambda^n) < 0\}$; i.e., $\{\lambda^n\}$ oscillates on both sides of $\lambda^*$. Then there must be infinite number of $\lambda^n$'s on at least one side of $\lambda^*$. There are two subcases:

(1) Infinite number of $\lambda^n$'s occur only on one side of $\lambda^*$. The contradiction can be shown using the same method presented in Case 1.

(2) Infinite numbers of $\lambda^n$'s occur on both sides of $\lambda^*$. We denote $\{\lambda_L^n\}$ as the $\lambda^n$'s on the left side of $\lambda^*$ and $\{\lambda_R^n\}$ as those on the right side. We define $\tilde{\lambda}_L = \sup\{\lambda_L^n\}$ and $\tilde{\lambda}_R = \inf\{\lambda_R^n\}$ as shown in Fig. C1b, where $V(\tilde{\lambda}_L) = \kappa_L$ and $V(\tilde{\lambda}_R) = \kappa_R$. We have:

$$\lim_{n\to\infty} V(\lambda_L^n) = \kappa_L > 0 \tag{C4}$$

$$\lim_{n\to\infty} \lambda_L^n = \tilde{\lambda}_L \tag{C5}$$

$$\lim_{n\to\infty} V(\lambda_R^n) = \kappa_R < 0 \tag{C6}$$

$$\lim_{n\to\infty} \lambda_R^n = \tilde{\lambda}_R \tag{C7}$$

And,

$$\lim_{n\to\infty} \lambda^{n+1} = \lim_{n\to\infty} (\lambda^n - V(\lambda^n)\frac{\lambda^n - \lambda^{n-1}}{V(\lambda^n) - V(\lambda^{n-1})}) \in (\tilde{\lambda}_L, \tilde{\lambda}_R) \tag{C8}$$

Contradiction! ■

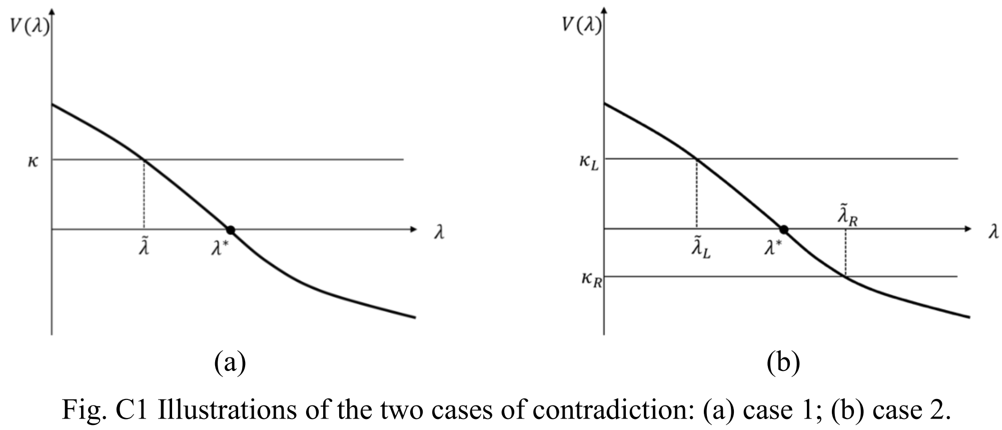


Fig. C1 Illustrations of the two cases of contradiction: (a) case 1; (b) case 2.

**Appendix D. The dynamic programming approach to the segment-level problem**

We first reproduced the dynamic programming method used by Lee and Madanat (2014a, 2015) with only minor modifications to solve subproblem 1. To this end, we assume that the maintenance intensity $v_{k\tau}$ and the pavement roughness level $s_k(\tau)$ take values form predefined discrete sets, i.e., $v_{k\tau} \in \left\{0, \frac{1}{d}D_{k\tau}, \frac{2}{d}D_{k\tau}, \cdots, D_{k\tau}\right\}$ for $\tau \in \{1, \cdots, T_k\}$, and $s_k(\tau) \in M_{k\tau} = \left\{s_k^{new}, s_k^{new} + \frac{\bar{s}_k(\tau) - s_k^{new}}{N}, s_k^{new} + 2\frac{\bar{s}_k(\tau) - s_k^{new}}{N}, \cdots, \bar{s}_k(\tau)\right\}$ for $\tau \in \{0, \cdots, T_k\}$, where $\bar{s}_k(\tau)$ is the maximum allowed roughness for segment $k$ in year $\tau$; i.e., if $s_k(\tau) > \bar{s}_k(\tau)$, the roughness level would exceed $s_k^{max}$ in the following year $\tau + 1$. There are $d + 2$ decision options in each year $\tau \in \{1, \cdots, T_k\}$: do nothing; rehabilitation only; and maintenance only with intensity $\frac{1}{d}D_{k\tau}, \frac{2}{d}D_{k\tau}, \cdots, D_{k\tau}$, respectively. Let $Y_k(q_{k\tau})$ denote the cost-to-go in year $\tau$ (i.e. the minimum total discounted cost from year $\tau$ to $T_k$), the algorithm is described as follows:

*Step 1.* For each $T_k \in \{T_k^{min}, \cdots, T_k^{max}\}$, set the boundary condition as $Y_k\left(q_{k,T_k}\right) = 0,\ \forall\ q_{k,T_k}$. For each year $\tau = T_k - 1, T_k - 2, \cdots, 0$, $s_k(\tau) \in M_{k\tau}$ and $h_{k\tau} = h_{k0} + \tau$, we generate $Y_k(q_{k\tau})$ in the backward direction by the Bellman equation:

$$Y_k(q_{k\tau}) = \min_{x_{k\tau,1}, x_{k\tau,2}, v_{k\tau}} \left\{\int_\tau^{\tau+1} l_k\left(c_k^1 s_k(u) + c_k^2\right) e^{-ru} du + x_{k\tau,1}\left(\gamma_k^1 v_{k\tau} + \gamma_k^2\right) e^{-r\tau} + x_{k\tau,2}(m_k^1 R_{k\tau} + m_k^2) e^{-r\tau} + Y_k(q_{k,\tau+1})\right\} \quad \text{(D1)}$$

where

$$q_{k,\tau+1} = \{s_k(\tau + 1), h_{k,\tau+1}\} = \left\{F_k\left(s_k(\tau) - x_{k\tau,2} G_k\left(R_{k\tau}, s_k(\tau)\right), 1, h_{k\tau}, \bar{b}_k - x_{k\tau,1} E_k(v_{k\tau}, s_k(\tau))\right), h_{k\tau} + 1\right\} \quad \text{(D2)}$$

$$Y_k\left(q_{k,\tau+1}\right) = \frac{s_k - s_k(\tau+1)}{s_k - s_k'} Y_k\left(s_k', h_{k,\tau+1}\right) + \frac{s_k(\tau+1) - s_k'}{s_k - s_k'} Y_k\left(s_k, h_{k,\tau+1}\right) \quad \text{(D3)}$$

$s_k'$ and $s_k$ are the two consecutive roughness indices in $M_{k,\tau+1}$ that satisfy $s_k' \le s_k(\tau + 1) \le s_k$.

*Step 2.* For each year $\tau = 0, 1, \cdots, T_k - 1$, record the optimal decision in the forward direction:

$$\left(x_{k\tau,1}^*, x_{k\tau,2}^*, v_{k\tau}^*\right) = \underset{x_{k\tau,1}, x_{k\tau,2}, v_{k\tau}}{\operatorname{argmin}} \left\{\int_\tau^{\tau+1} l_k\left(c_k^1 s_k(u) + c_k^2\right) e^{-ru} du + x_{k\tau,1}\left(\gamma_k^1 v_{k\tau} + \gamma_k^2\right) e^{-r\tau} + x_{k\tau,2}(m_k^1 R_{k\tau} + m_k^2) e^{-r\tau} + Y_k\left(q_{k,\tau+1}\right)\right\} \quad \text{(D4)}$$

where $q_{k0} = \{s_k(0), h_{k0}\}$.

*Step 3.* Find the $T_k$ that minimizes $\frac{Z_k^S}{1 - e^{-rT_k}}$.

In the first step, we apply the Bellman equation (D1) recursively to generate $Y_k(q_{k\tau})$ for all $\tau \in \{0, \cdots, T_k - 1\}, s_k(\tau) \in M_{k\tau}$ and $h_{k\tau} = h_{k0} + \tau$. The pavement state in year $\tau + 1$ is calculated by equation (D2). The cost-to-go $Y_k(q_{k,\tau+1})$ is approximately by linear interpolation between $Y_k(s'_k, h_{k,\tau+1})$ and $Y_k(s_k, h_{k,\tau+1})$; see equation (D3). The optimal decision $(x^*_{k\tau,1}, x^*_{k\tau,2}, v^*_{k\tau})$ that minimizes $Y_k(q_{k\tau})$ in each year $\tau$ is obtained and recorded in step 2 with the initial state $q_{k0}$.

To solve subproblem 2, we make the following changes to the above algorithm: i) in year 0 there are $d + 2$ decisions as in other years; and ii) $T_k$ is replaced by $t^T_k$, whose range is $\left[T^{min'}_k, T^{max'}_k\right]$, where $T^{min'}_k = \max\{0, T^{min}_k - h_{k0}\}$ and $T^{max'}_k = \max\{0, T^{max}_k - h_{k0}\}$. Finally, we choose the $t^T_k$ that minimizes equation (12).

## References


ASCE, 2017. 2017 infrastructure report card: A comprehensive assessment of America's Infrastructure. (accessed on May 2, 2017). American Society of Civil Engineers, Reston, Virginia. http://www.infrastructurereportcard.org/wp-content/uploads/2016/10/2017-Infrastructure-Report-Card.pdf

Bai, Y., Gungor, O.E., Hernandez-Urrea, J.A., Ouyang, Y., Al-Qadi, I.L., 2015. Optimal pavement design and rehabilitation planning using a mechanistic-empirical approach. EURO Journal on Transportation and Logistics 4(1), 57-73.

Bellman, R.A., 1957. Markovian decision process. No. P-1066. Rand Corp., Santa Monica, CA.

Blum, C., Roli, A. 2003. Metaheuristics in combinatorial optimization: Overview and conceptual comparison. ACM Computing Surveys 35(3), 268-308.

Carnahan, J.V., Davis, W.J., Shahin, M.Y., Kean, P.L., Wu, M.I., 1987. Optimal maintenance decisions for pavement management. ASCE Journal of Transportation Engineering 113 (5), 554-572.

CBO, 2016. Approaches to make federal highway spending more productive (accessed on May 2, 2017). Congressional Budget Office of the United States, Washington, DC. www.cbo.gov/sites/default/files/114th-congress-2015-2016/reports/50150-Federal_Highway_Spending.pdf.

Chan, W., Fwa, T., Tan, C., 1994. Road maintenance planning using genetic algorithms. I: Formulation. Journal of Transportation Engineering 120(5), 693-709.

Chong, G.J., 1989. Rout and seal cracks in flexible pavement – A cost-effective preventive maintenance procedure. Rep. No. 890412, Ontario Ministry of Transportation, Ontario, Canada.

Chu, J., Chen, Y., 2012. Optimal threshold-based network-level transportation infrastructure life-cycle management with heterogeneous maintenance actions. Transportation Research Part B 46(9), 1123-1143.

Deshpande, V.P., Damnjanovic, I.D., Gardoni, P., 2010. Reliability- based optimization models for scheduling pavement rehabilitation. Computer-Aided Civil and Infrastructure Engineering 25, 227-237.

Durango-Cohen, P., 2007. A time series analysis framework for transportation infrastructure management. Transportation Research Part B 41 (5), 493-505.

Durango-Cohen, P., Sarutipand, P., 2007. Capturing interdependencies and heterogeneity in the management of multifacility transportation infrastructure system. Journal of Infrastructure Systems 13 (2), 115-123.

Fernandez, J., Friesz, T., 1981. Influence of demand-quality interrelationships on optimal policies for stage construction of transportation facilities. Transportation Science 15(1), 16-31.

Friesz, T., Fernandez, J., 1979. A model of optimal transport maintenance with demand responsiveness. Transportation Research Part B 13(4), 317-339.

Fwa, T., Chan, W., Tan, C., 1996. Genetic-algorithm programming of road maintenance and rehabilitation. Journal of Transportation Engineering 122(3), 246-253.

Fwa, T., Tan, C., Chan, W., 1994. Road maintenance planning using genetic algorithms. I: Analysis. Journal of Transportation Engineering 120(5), 710-722.

Golabi, K., Kulkarni, R., Way, G., 1982. A statewide pavement management system. Interfaces 12 (6), 5-21.

Gu, W., Ouyang, Y., Madanat, S., 2012. Joint optimization of pavement maintenance and resurfacing planning. Transportation Research Part B 46 (4), 511-519.

Hajibabai, L., Bai, Y., Ouyang, Y., 2014. Joint optimization of freight facility location and pavement infrastructure rehabilitation under network traffic equilibrium. Transportation Research Part B 63, 38-52.

Jorge, N., Stephen, J.W., 2006. Numerical Optimization, 286-290.

Kuhn, K., Madanat, S., 2005. Model uncertainty and the management of a system of infrastructure facilities. Transportation Research Part C13 (5), 391-404.

Labi, S., Sinha, K.C., 2003. The effectiveness of maintenance and its impact on capital expenditures. Indiana Department of Transportation.

Lee, J., Madanat, S., 2014a. Jointly optimal policies for pavement maintenance, resurfacing and reconstruction. EURO Journal on Transportation and Logistics, 4(1), 75-95.

Lee, J., Madanat, S., 2014b. Joint optimization of pavement design, resurfacing and maintenance strategies with history-dependent deterioration models. Transportation Research Part B 68, 141-153.

Lee, J., Madanat, S., 2015. A joint bottom-up solution methodology for system-level pavement rehabilitation and reconstruction. Transportation Research Part B 78, 106-122.

Lee, J., Madanat, S., Reger, D., 2016. Pavement systems reconstruction and resurfacing policies for minimization of life-cycle costs under greenhouse gas emissions constraints. Transportation Research Part B 93, 618-630.

Li, Y., Madanat, S., 2002. A steady-state solution for the optimal pavement resurfacing problem. Transportation Research Part A 36 (6), 525-535.

Madanat, S., 1993. Incorporating inspection decisions in pavement management. Transportation Research Part B 27 (6), 425-438.

Madanat, S., Ben-Akiva, M., 1994. Optimal inspection and repair policies for infrastructure facilities. Transportation Science 28 (1), 55-62.

Mamlouk, M.S., Dosa, M., 2014. Verification of effectiveness of chip seal as a pavement preventive maintenance treatment using LTPP data. International Journal of Pavement Engineering 15 (10): 10, 879-888.

Markow, M., Balta, W., 1985. Optimal rehabilitation frequencies for highway pavements. Transportation Research Record 1035, 31-43.

Miyamoto A., Kawamura K., Nakamura H., 2000. Bridge Management System and Maintenance Optimization for Existing Bridges. Computer-Aided Civil and Infrastructure Engineering 15, 45-55.

Ouyang, Y., 2007. Pavement resurfacing planning on highway networks: A parametric policy iteration approach. Journal of Infrastructure Systems (ASCE) 13(1), 65-71.

Ouyang, Y., Madanat, S., 2004. Optimal scheduling of rehabilitation activities for multiple pavement facilities: exact and approximate solutions. Transportation Research Part A 38, 347-365.

Ouyang, Y., Madanat, S., 2006. An analytical solution for the finite-horizon pavement resurfacing planning problem. Transportation Research Part B 40 (9), 767-778.

Peshkin, D.G., Hoerner, T.E., Zimmerman, K.A., 2004. Optimal timing of pavement preventive maintenance treatment applications. Washington, DC: Transportation Research Board, Publication NCHRP 523.

Ponniah, J.E., Kennepohl, G.J., 1996. Crack sealing in flexible pavements: a life-cycle cost analysis. Transportation Research Record 1529, 86-94.

Rashid, M.M., Tsunokawa K., 2012. Trend curve optimal control model for optimizing pavement maintenance strategies consisting of various treatments. Computer-Aided Civil and Infrastructure Engineering, 27(3), 155-169.

Sathaye, N., Madanat, S., 2011. A bottom-up solution for the multi-facility optimal pavement resurfacing problem. Transportation Research Pare B 45 (7), 1004-1017.

Sathaye, N., Madanat, S., 2012. A bottom-up optimal pavement resurfacing solution approach for large-scale networks. Transportation Research Part B 46 (4), 520-528.

Tsunokawa, K., Hiep, D.V., Ul-Isalm, R., 2006. True optimization of pavement maintenance options with what-if models. Computer-Aided Civil and Infrastructure Engineering 21, 193-204.

Tsunokawa K., Ul-Isalm, R., 2002. Optimal strategies for highway pavement management in developing countries. Computer-Aided Civil and Infrastructure Engineering 17, 194-202.

Tsunokawa, K., Schofer, J., 1994. Trend curve optimal control model for highway pavement maintenance: case study and evaluation. Transportation Research Part A 28 (2), 151-166.